THE UNIVERSITY OF ALBERTA

TRANSIENT ANALYSIS OF TRANSMISSION LINES

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Transient Analysis of Transmission Lines" submitted by Ferial M.E. El-Hawary (El-Bibany) in partial fulfilment of the requirements for the degree of Master of Science.

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ABSTRACT

In this thesis, a method for the evaluation of the transient response of a nonuniform lossy transmission line is presented. The method of characteristics is employed to obtain difference equations describing the transmission line.

A stepped line approximation is used to analyze the transient response of the given nonuniform line. The concept of electrical length is employed in dividing the line into a number of equal delay sections. The set of difference equations describing the set stepped line is suitable for digital computer solution.

The computational procedure involving the use of a digital computer is illustrated for a specific distributions of L(x), C(x), r(x) and g(x).

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CHAPTER-I

Introduction

1.1 Background

In the design of all apparatus and machines related to the generation and transmission of electrical power, it is important to know the salient features of the overvoltages likely to be encountered. Faults caused by switching are becoming more important with the ever higher transmission voltages being used.

Further the basic form of implementing computer hardware is in mounting integrated circuits (I.C.) packages on multilayer printed circuit boards which in turn are mounted on so called mother boards. The pins of the I.C. packages on the printed circuit board are usually connected to each other by etched metal strips, and circuit engineers are forced to regard these metal strips as a length of a transmission line as the speed of the circuit approaches the nanoseconds range[8].

Furthermore, the problem of constructing a mathematical model for simulating the electrical properties of memory arrays has been considered in 1963 by Weeks,[11] who found that a memory array can be approximated by a system of linear transmission lines.

Because of the above considerations, transient analysis of general transmission lines has recently become the object of

wide-spread interest of many investigators. The analysis of transient waves on a transmission line leads to a set of partial differential equations which define the relation between current and voltage as functions of time and position on the line. In general, these equations do not lend themselves to a closed form solution except in the special case of distortionless lines (where $\frac{R}{L} = G/C$), which includes the lossless case ($R = G = 0$). Thus using numerical techniques for solving such equations is inevitable

The application of the method of characteristics was shown in 1967 by F.H. Branin Jr., [1] to provide a simple analytic solution for the problem of a uniform lossless line. The method was shown to be superior in both speed and accuracy to the more familiar method of integrating the differential equations that describe a lumped L.C. model of the line. Branin did not attempt in this work to extend the method to more general lines.

Y.K. Liu,[2] in 1968, considered the transient analysis of a uniform lossy line terminated in a resistive load. For his analysis he used the method of characteristics and a second order Runge-Kutta technique to solve numerically the resulting pair of ordinary differential equations.

Transform methods for analyzing the transient response of uniform lines was considered by K.A. Chen in the same year.[3]

Chen's work was concerned with interconnected lines as an aid in the design of memory arrays in digital computer hardware.

The investigation carried out by Wassel[4] used transform methods for the analysis which was concerned with the effects of RC loadings of pulse signal transmission lines. It is noted that R. Murray-Shelley[5] investigated the same problem in the same year using the method of characteristics (often called the graphical method). In 1969, Y.K. Liu,[6] reported the application of the same technique he used in (2) when the line is terminated in a tunnel diode.

In 1970, V. Dvorak[9] reported a novel method of treating this problem. He used the method of characteristics in a way that is straight forward and avoids entirely lengthy numerical techniques such as Runge-Kutta method. One of the advantages of Dvorak's method is that any loading configuration can be handled easily. Further the treatment of nonuniform lines was shown to be best analyzed using a stepped line approximating the original line, that is the division of the line into a number of sections having the same delay (electrical length).

1.2 Scope of the thesis

In this thesis the application of the method of characteristics to the problem of computing the transient response of a general transmission line is presented. The analysis for a nonuniform lossless line is presented. The method of analyzing both uniform and nonuniform lossy lines is discussed, here a stepped line approximating the given line is used. The choice of proper time step and

number of sections of the stepped line is considered for the case of uniform lines. This is to some extent an extension of the work of V. Dvorak.[9]

A general transmission line transient analysis computer program is described. The program is written in FORTRAN IV programming language. The program is discussed in some detail and numerical results are given for some example transmission lines.

CHAPTER 2

Transient analysis of lossless nonuniform transmission lines

2.1 The method of characteristics

The set of partial differential equations describing a uniform transmission line is,

$$-\frac{\partial V}{\partial Z} = L\frac{\partial i}{\partial t} + R\ i \tag{2.1}$$

$$-\frac{\partial i}{\partial Z} = C\frac{\partial V}{\partial t} + G\ V \tag{2.2}$$

where L, C, R and G are inductance, capacitance, resistance and conductance per unit length of the line respectively, $V = V(Z,t)$ and $i = i(Z,t)$ are the voltage and current, respectively, at distance Z from one end of the transmission line at time t. In essence the method of characteristics transforms (2.1) and (2.2) into two ordinary differential equations as follows. Let

$$\phi_1 = Ri + Li_t + V_Z = 0 \tag{2.3}$$

$$\phi_2 = GV + CV_t + i_Z = 0 \tag{2.4}$$

where

$$V_Z = \frac{\partial V}{\partial Z}$$

$$V_t = \frac{\partial V}{\partial t}$$

$$i_Z = \frac{\partial i}{\partial Z}$$

$$i_t = \frac{\partial i}{\partial t}$$

Defining

$$\phi = \phi_1 + \lambda\ \phi_2 = 0$$

Then

$$\phi = Ri + \lambda\ GV + L[i_t + \frac{\lambda}{L} i_Z] + \lambda\ C[V_t + \frac{V_Z}{\lambda C}] = 0 \qquad (2.5)$$

This is a voltage equation so that the quantity between brackets in the third term of (2.5) should be equal to the total derivative of the current $\frac{di}{dt}$, hence we have

$$\frac{di}{dt} = i_t + \frac{\lambda}{L}\ i_Z$$

but

$$\frac{di}{dt} = i_t + \frac{\partial i}{\partial Z}\ \frac{dZ}{dt}$$

hence

$$\frac{dZ}{dt} = \frac{\lambda}{L} \qquad (2.6)$$

Similarly the quantity between brackets in the last term of (2.5) should be equal to the total derivative of the voltage $\frac{dV}{dt}$, hence

$$\frac{dZ}{dt} = \frac{1}{\lambda C} \tag{2.7}$$

by (2.6) and (2.7) we have

$$\lambda = \pm\sqrt{\frac{L}{C}} \tag{2.8}$$

or

$$\frac{dZ}{dt} = \pm \frac{1}{\sqrt{LC}} \tag{2.9}$$

Substituting (2.8) into (2.5)

$$\phi = Ri \pm GV\sqrt{\frac{L}{C}} + L \frac{di}{dt} \pm \sqrt{LC} \frac{dV}{dt} = 0$$

or

$$\frac{d}{dt} [V \pm\sqrt{\frac{L}{C}} i] = - [\frac{G}{C} V \pm \frac{R}{\sqrt{LC}} i] \tag{2.10}$$

Now (2.9) implies that in the (Z-t) plane we have essentially straight line characteristic curves in the case of a uniform transmission line.

2.2 The stepped line approximation

Consider a lossless transmission line characterized by distributed inductance L(Z) > 0 and capacitance C(Z) > 0 per unit length, where Z is the physical position on the line. The electrical

position along the line is defined by:

$$y(Z) = \int_0^Z \sqrt{L(\eta)\, C(\eta)}\; d\eta \tag{2.11}$$

and the local characteristic impedance by

$$\rho(y) = \sqrt{L[Z(y)] / C[Z(y)]} \tag{2.12}$$

Now the characteristic curves in the (Z-t) plane are not straight lines in the case of nonuniform transmission lines, but the introduction of the electrical position y(Z) yields straight line characteristic curves in the (y-t) plane.

A transmission line of total length h may be divided into n sections of the same delay $\Delta y = \frac{y(h)}{n}$. If n is chosen large enough, then any such section can be approximated by a uniform lossless line with an average characteristic impedance ρ_K $(1 \leq K \leq n)$ and with total propagation delay $T = \Delta y$. The resulting cascade of n uniform lossless terminating networks is described by 2 n difference equations in the time domain of the form given in (A-9) and (A-10), plus the two equations describing the terminating networks. A stepped line approximating the given nonuniform line is shown in figure 2.1.

The basic equations for a uniform lossless line are derived in Appendix A. Applying (A-9) to the incremental line between the $(K-1)^{st}$ and K^{th} nodes in figure 2.1 yields

$$V_K(t) + \rho_{K-1}\, i_K(t) = V_{K-1}(t-T) + \rho_{K-1}\, i_{K-1}(t-T) \tag{2.13}$$

$$2 \leq K \leq n+1$$

where we substituted

$$V_K(t) = V(1,t)$$

$$V_{K-1}\ (t) = V(0,t)$$

Further (A-10) is applied to the incremental line between the K^{th} and $(K-1)^{st}$ nodes giving

$$V_K(t) - \rho_K\, i_K(t) = V_{K+1}\ (t-T) - \rho_K\, i_{K+1}\ (t-T) \qquad (2.14)$$

$$1 \leqslant K \leqslant n$$

The terminating networks A and B are given by their V-A characteristics

$$A\ :\ i_1 = f_1\ (V_1) \qquad (2.15)$$

$$B\ :\ i_{n+1} = f_2\ (V_{n+1}) \qquad (2.16)$$

Given the initial conditions at the time instant t=0, the values of V_K (t) and i_K (t) can then be computed at time instants $t = mT$, $1 \leqslant m \leqslant m_{max}$.

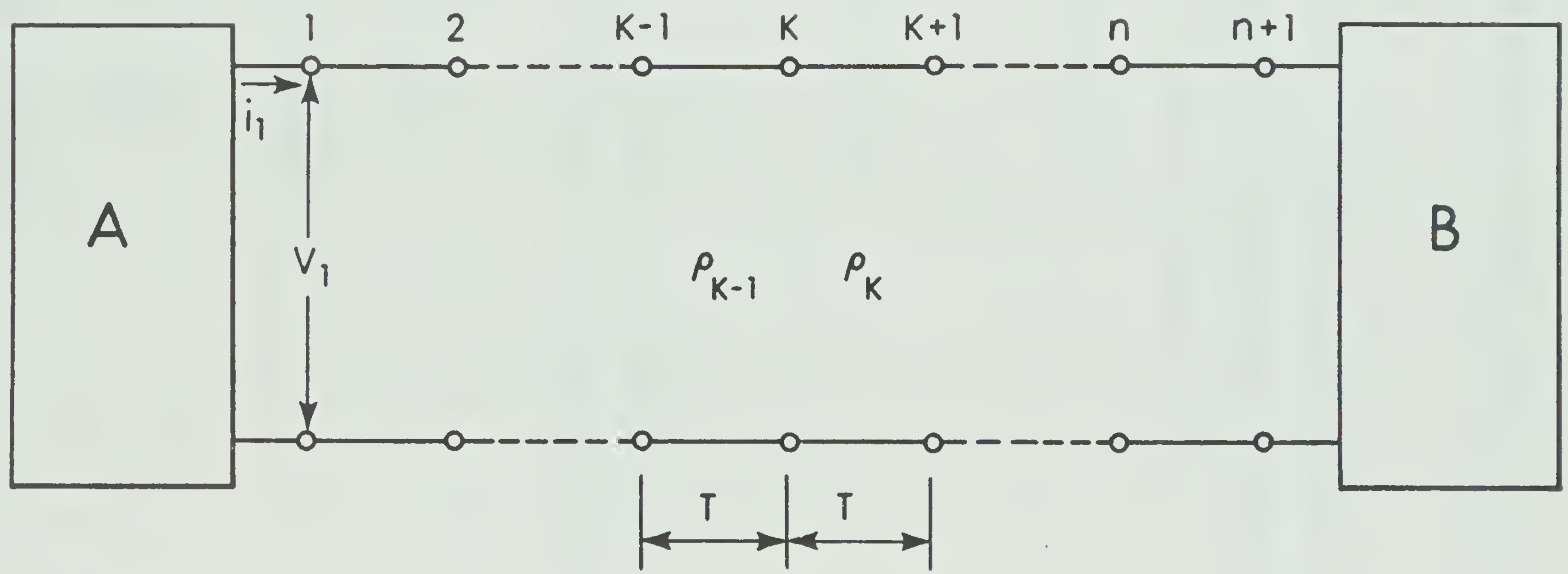


FIG. 2.1 A STEPPED LINE

CHAPTER 3

Transient analysis of lossy uniform lines

3.1 Analysis

Consider a lossy uniform line for which (2.9) implies that in the (Z-t) plane we have essentially straight line characteristic curves as shown in figure (3.1).

where along the α curve we have

$$\frac{dZ}{dt} = + \frac{1}{\sqrt{LC}}$$

while along the β curve we have

$$\frac{dZ}{dt} = - \frac{1}{\sqrt{LC}}$$

Using (2.10) we have the following two differential equations each being defined along a different characteristic direction in the (Z-t) plane

Along the α curve: $\frac{dZ}{dt} = \frac{1}{\sqrt{LC}}$

$$\frac{d}{dt}[V+\rho i] = -\frac{1}{\sqrt{LC}}[\rho GV+Ri] \tag{3.1}$$

Along the β curve : $\frac{dZ}{dt} = \frac{-1}{\sqrt{LC}}$

$$\frac{d}{dt}[V-\rho i] = -\frac{1}{\sqrt{LC}}[\rho GV-Ri] \tag{3.2}$$

Note that (3.1) defines a forward propagating wave where as (3.2) defines a backward propagating wave.

Dividing the line into n sections of delay

$$T = h\sqrt{LC}\ /\ n \tag{3.3}$$

and taking $dt = \Delta t = T$, one gets

$$\Delta[V\ (Z,t) + \rho i\ (Z,t)]\Big|_{K}^{K+1} = \frac{-h}{n}\,[G\,\rho\,V\ (Z,t)$$

$$+ R\ i\ (Z,t)] \tag{3.4}$$

$$\Delta[V\ (Z,t) - \rho i\ (Z,t)]\Big|_{K+1}^{K} = \frac{-h}{n}\,[G\,\rho\,V\ (Z,t)$$

$$- R\ i\ (Z,t)] \tag{3.5}$$

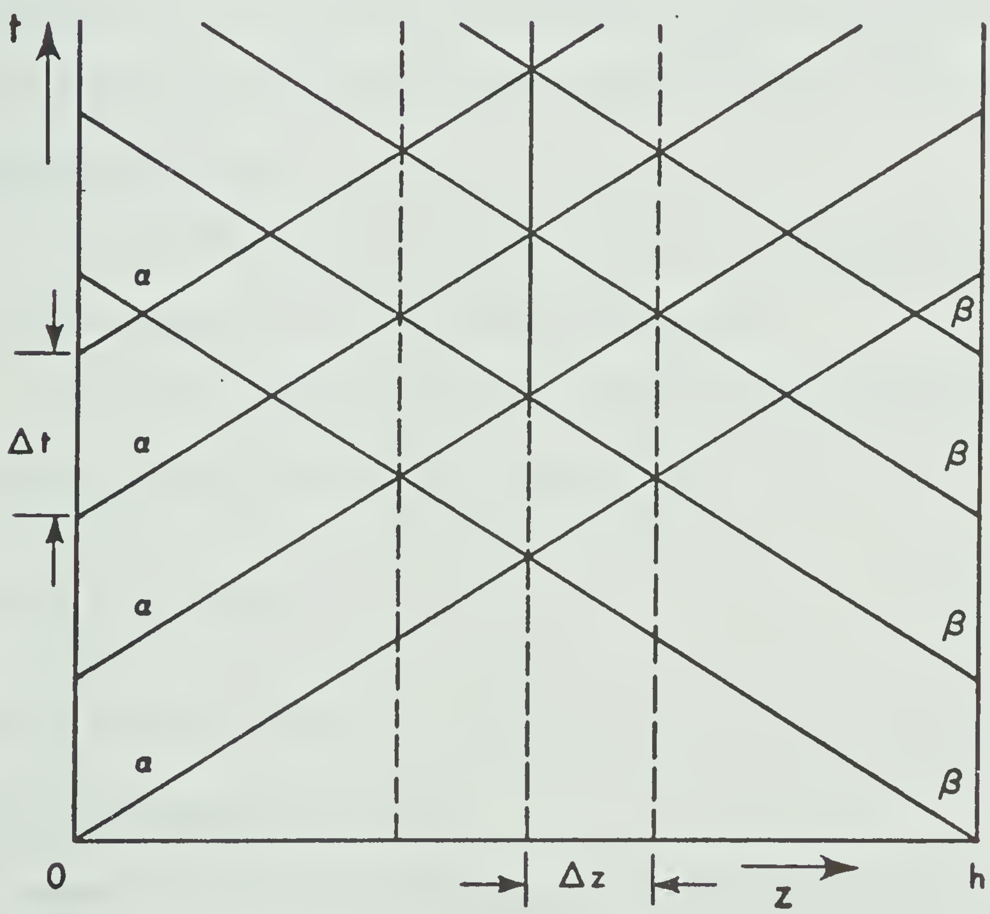


FIG. 3.1 FAMILIES OF α AND β CHARACTERISTIC CURVES IN Z-t PLANE

letting

$$g = \frac{Gh}{n}, \quad r = \frac{Rh}{n} \tag{3.6}$$

Then (3.4) and (3.5) become

$$\Delta[V\ (Z,t) + \rho\ i\ (Z,t)]\Big|_{K}^{K+1} = -\ [g\ \rho\ V\ (Z,t)$$

$$+\ r\ i\ (Z,t)] \tag{3.7}$$

$$\Delta[V\ (Z,t) - \rho\ i\ (Z,t)]\Big|_{K+1}^{K} = -\ [g\ \rho\ V\ (Z,t)$$

$$-\ r\ i\ (Z,t)] \tag{3.8}$$

Notice that any event occuring at the K^{th} node at the time instant (t-T) arrives at the $(K+1)^{st}$ node at the time instant, t, for the forward propagating wave, (the reverse is true for backward propagating wave).

So that writing (3.7) and (3.8) in a discrete (difference equation) form leads to the following question:

how are we going to discretize the right hand side of each of (3.7) and (3.8)? To this end we let

$$f(Z,t) = g\ \rho\ V(Z,t) + r\ i\ (Z,t) \tag{3.9}$$

$$g(Z,t) = g\ \rho\ V\ (Z,t) - r\ i(Z,t) \tag{3.10}$$

We have the following two cases

I. If we assume that f(Z,t) and g(Z,t) remains constant over the time interval [t-T, t], this is equivalent to the assumption

$$f(Z,t) = f_K(t-T) \tag{3.11}$$

$$g(Z,t) = g_{K+1}(t-T) \tag{3.12}$$

II. If we assume that $f(Z,t)$ and $g(Z,t)$ change in a linear fashion over the time interval $[t-T,t]$ then

$$f(Z,t) = f_K(t-T) + \frac{f_{K+1}(t) - f_K(t-T)}{2}$$

or

$$f(Z,t) = [f_K(t-T) + f_{K+1}(t)] / 2 \tag{3.13}$$

and

$$g(Z,t) = [g_{K+1}(t-T) + g_K(t)] / 2 \tag{3.14}$$

Application of (3.11) and (3.12) to (3.7) and (3.8) respectively yields

$$V_{K+1}(t) + \rho\ i_{K+1}(t) = [1 - \rho g]\ V_K(t-T) +$$

$$[\rho - r]\ i_K(t-T) \tag{3.15}$$

$$V_K(t) - \rho\ i_K(t) = [1 - \rho g]\ V_{K+1}(t-T) -$$

$$[\rho - r]\ i_{K+1}(t-T) \tag{3.16}$$

Equations (3.15) and (3.16)will be referred to as the first formulation equations.

Further, application of (3.13) and (3.14) to (3.7) and (3.8) respectively yields

$$V_{K+1}(t)\ [1 + \frac{\rho g}{2}] + i_{K+1}(t)\ [\rho + \frac{r}{2}] =$$

$$V_K(t-T)\ [1 - \frac{\rho g}{2}] + i_K(t-T)\ [\rho - \frac{r}{2}] \qquad (3.17)$$

$$V_K(t)\ [1 + \frac{\rho g}{2}] - i_K(t)\ [\rho + \frac{r}{2}] =$$

$$V_{K+1}(t-T)\ [1 - \frac{\rho g}{2}] - i_{K+1}(t-T)\ [\rho - \frac{r}{2}] \qquad (3.18)$$

Equations (3.17) and (3.18)will be referred to as the second formulation equations.

The first and second formulation equations are the difference equations of a very short lossy line in the time domain.

3.2 Boundary conditions

Let the line have the following initial conditions

$$i_K(0) = 0 \qquad 1 \le K \le n + 1 \qquad (3.19)$$

$$V_K(0) = 0 \qquad 1 \le K \le n + 1 \qquad (3.20)$$

Let the sending end terminating network be an ideal current source whose current output is a prespecified function of time I(t). If the source is assumed to have a shunt conductance G_s, then the following equation holds true.

$$i_1(t) = I(t) - G_s\ V_1(t) \qquad (3.21)$$

If the line is terminated in a load resistance R_r then

$$V_{n+1}(t) = R_r\ i_{n+1}(t) \qquad (3.22)$$

The line with its terminating network is shown in figure (3.2).

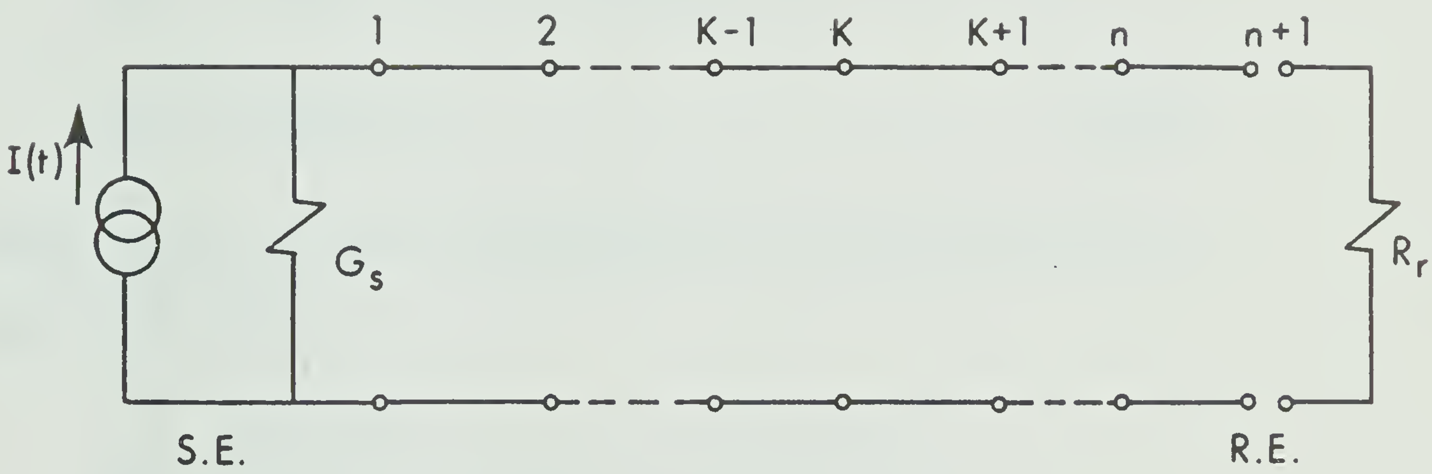


FIG. 3.2 UNIFORM TRANSMISSION LINE WITH

ITS TERMINATING NETWORKS

3.3 Implementing the first formulation

We rewrite the first formulation equations (3.15)and (3.16) as

$$V_K(t) + \rho\, i_K(t) = [1 - \rho g]\, V_{K-1}[t-T] + [\rho - r]\, i_{K-1}(t-T) \qquad (3.23)$$

$$V_K(t) - \rho\, i_K(t) = [1 - \rho g]\, V_{K+1}(t-T) - [\rho - r]\, i_{K+1}(t-T) \qquad (3.24)$$

Adding (3.24) to (3.23) and subtracting (3.24) from (3.23) respectively yields

$$V_K(t) = \frac{(1 - \rho g)}{2}\, [V_{K-1}(t-T) + V_{K+1}(t-T)] + \frac{(\rho - r)}{2}\, [i_{K-1}(t-T) - i_{K+1}(t-T)] \quad 2 \leq K \leq (n-1) \qquad (3.25)$$

$$i_K(t) = \frac{(1 - \rho g)}{2\rho}\, [V_{K-1}(t-T) - V_{K+1}(t-T)] + \tfrac{1}{2}[1 - \frac{r}{\rho}]\, [i_{K-1}(t-T) + i_{K+1}(t-T)] \quad 2 \leq K \leq (n-1) \qquad (3.26)$$

Thus the values of $V_K(t)$ and $i_K(t)$ at times $t = mT$, $1 \leq m \leq m_{max}$ can be computed using (3.23), (3.24), (3.25)&(3.26).

3.4 Implementing the second formulation

Rewriting the second formulation equations (3.17) and (3.18) as

$$V_K(t)\ [1 + \frac{\rho g}{2}] + i_K(t)\ [\rho + \frac{r}{2}] = V_{K-1}(t-T)[1 - \frac{\rho g}{2}]$$

$$+ i_{K-1}(t-T)\ [\rho - \frac{r}{2}] \qquad (3.27)$$

$$V_K(t)\ [1 + \frac{\rho g}{2}] - i_K(t)\ [\rho + \frac{r}{2}] = V_{K+1}(t-T)[1 - \frac{\rho g}{2}]$$

$$- i_{K+1}(t-T)\ [\rho - \frac{r}{2}] \qquad (3.28)$$

Adding (3.27) and (3.28) and subtracting (3.28) from (3.27) respectively yields

$$V_K(t) = \Big\{[V_{K-1}(t-T) + V_{K+1}(t-T)]\ [1 - \frac{\rho g}{2}] +$$

$$[i_{K-1}(t-T) - i_{K+1}(t-T)]\ [\rho - \frac{r}{2}]\Big\} \Big/ (2 + \rho g) \qquad (3.29)$$

$$i_K(t) = \Big\{[V_{K-1}(t-T) - V_{K+1}(t-T)]\ [1 - \frac{\rho g}{2}] +$$

$$[i_{K-1}(t-T) + i_{K+1}(t-T)]\ [\rho - \frac{r}{2}]\Big\} \Big/ (2\rho + r) \qquad (3.30)$$

Thus the values of $V_K(t)$ and $i_K(t)$ at instants $t = mT$, $1 \leqslant m \leqslant m_{max}$ can be computed using equations (3.19), (3.20), (3.29), (3.30), (3.21), (3.22).

3.5 Error estimation

If we let

$$W(Z,t) = V(Z,t) + \rho\, i(Z,t) \tag{3.31}$$

$$f(Z,t) = \frac{-1}{\sqrt{LC}} [G\, \rho\, V(Z,t) + R\, i(Z,t)] \tag{3.32}$$

Then equation (3.1) is transformed to

$$\frac{dW}{dt} = f(t) \tag{3.33}$$

for a specific position Z.

Thus the increment of change of W, which is denoted by ΔW over the interval $[t_o, t_o + \Delta t]$ is

$$\Delta W = [f(t)]_{avg} \cdot \Delta t \tag{3.34}$$

Where $[f(t)]_{avg}$ is the average value of f(t) in the interval $[t_o, t_o + \Delta t]$ given by

$$[f(t)]_{avg} = \frac{1}{\Delta t} \int_{t_o}^{t_o + \Delta t} f(\S)\, d\, \S \tag{3.35}$$

Now using Taylor's expansion for the right hand side of (3.35) one gets

$$[f(t)]_{avg} = f(t_o) + \frac{\Delta t}{2!} \dot{f}(t) \Big|_{t_o} +$$

$$\frac{(\Delta t)^2}{3!} \ddot{f}(t) \Big|_{t_o} + \ldots + \frac{(\Delta t)^n}{(n+1)!} f^n \Big|_{t_o} + \ldots \qquad (3.36)$$

If Δt is sufficiently small then

$$[f(t)]_{avg} = f(t_o) + \frac{\Delta t}{2!} \dot{f}(t) \Big|_{t_o} + \frac{(\Delta t)^2}{3!} \ddot{f}(t) \Big|_{t_o} +$$

$$\frac{(\Delta t)^3}{4!} \dddot{f}(t) \Big|_{t_o} \qquad (3.37)$$

Thus a sufficiently accurate value of ΔW is

$$\Delta W = \Delta[f_o + \frac{\Delta}{2} \dot{f}_o + \frac{\Delta^2}{6} \ddot{f}_o + \frac{\Delta^3}{24} \dddot{f}_o] \qquad (3.38)$$

where

$$f_o = f\ (t_o)$$

$$\dot{f}_o = \dot{f}\ (t) \Big|_{t_o}$$

and so on.

a - <u>First formulation</u>

Referring to (3.11), the average value of $f(t)$ over the interval $[t_o, t_o+\Delta]$ was taken to be $f(t_o)$, (where t_o=t-T, Δ=T in this case) thus the estimated increment of change in W, denoted by ΔW_{s1} is

$$\Delta W_{s1} = [f(t)]_{avg_1} \cdot \Delta t \qquad (3.39)$$

hence

$$\Delta W_{s_1} = f_o \cdot \Delta \tag{3.40}$$

This provides us with a sufficiently accurate error measure for the first formulation which we denote by ε_{s1}, so we have

$$\varepsilon_{s_1} = \left| \Delta W_{s_1} - \Delta W \right| \tag{3.41}$$

Substituting (3.38) and (3.40) into (3.41) we get

$$\varepsilon_{s_1} = \frac{\Delta^2}{2} \dot{f}_o + O_3 (\Delta t) \tag{3.42}$$

with $O_3(\Delta t)$ being terms in Δt higher than third order.

Now if an upper limit on the error incurred in implementing the first formulation is given by ε_a, then,

$$\varepsilon_a > \frac{\Delta^2}{2} \dot{f}_o$$

or

$$\Delta t < \left(\frac{2\varepsilon_a}{\dot{f}_o} \right)^{1/2} \tag{3.43}$$

which defines an upper limit on the value of the time step to be chosen so that the given accuracy is achieved. Let this upper limit be Δt_1, then

$$\Delta t_1 = \left(\frac{2\varepsilon_a}{\dot{f}_o} \right)^{1/2}$$

b - Second formulation

Referring to (3.13), the average value of f(t) over the interval $[t_o, t_o+\Delta]$ was taken to be

$$[f(t)]_{avg_2} = \tfrac{1}{2}[f_o + f_1] \tag{3.44}$$

where

$$f_1 = f(t_o + \Delta t) \tag{3.45}$$

thus the estimated increment of change in W denoted by ΔW_{s_2} is

$$\Delta W_{s_2} = [f(t)]_{avg_2} \cdot \Delta t \tag{3.46}$$

which yields the error measure for the 2nd formulation ε_{s_2} which is given by

$$\varepsilon_{s_2} = \mid \Delta W_{s_2} - \Delta W \mid \tag{3.47}$$

applying (3.38) and (3.46) to (3.47) yields

$$\varepsilon_{s_2} = \frac{\Delta^3}{12} \ddot{f}_o + 0_4 (\Delta t) \tag{3.48}$$

Now if an upper limit on the error incurred in implementing the 2nd formulation is given by ε_a then

$$\varepsilon_a > \frac{\Delta^3}{12} \ddot{f}_o$$

or

$$\Delta t < \left(\frac{12\varepsilon_a}{\ddot{f}_o} \right)^{1/3} \tag{3.49}$$

define

$$\Delta t_2 = \left(\frac{12\varepsilon_a}{\ddot{f}_o} \right)^{1/3}$$

This specifies the upper limit (Δt_2) on the value of the time step to be chosen, so that the given accuracy is achieved.

c - <u>Simplified calculation of $\ddot{f}_o$</u>

In appendix (B) it is shown that the function W obeys the following equation

$$W_{K+1} = W_K \exp[-\Delta t.s - \tfrac{1}{2}(\frac{r}{\rho} + g\rho)] \tag{3.50}$$

where s is the Laplace operator and hence the term $\Delta t.s$ in the exponent represents the time delay between two consecutive nodes.

We are concerned only with the deterioration in the wave W, not the delay, so that we need only to consider

$$W_{K+1} = W_K \exp[-\tfrac{1}{2}(\frac{r}{\rho} + g\rho)] \tag{3.51}$$

Let

$$\varepsilon = \exp[-\tfrac{1}{2}(\frac{r}{\rho} + g\rho)] \tag{3.52}$$

Then

$$W_{K+1} = \varepsilon\, W_K \tag{3.53}$$

$$W_{K+2} = \varepsilon^2\, W_K \tag{3.54}$$

$$W_{K+3} = \varepsilon^3\, W_K \tag{3.55}$$

This enables one to evaluate $\dot{f}_o$ and $\ddot{f}_o$ which are required in the calculation of upper limits on the time step in the previous section.

By definition

$$f(t) = \frac{dW}{dt} \tag{3.56}$$

Then in a discrete form one can write

$$f_o = \frac{W_{K+1} - W_K}{\Delta t} \tag{3.57}$$

Substituting (3.53) into (3.57) then

$$f_o = \frac{W_K}{\Delta t} (\varepsilon - 1) \tag{3.58}$$

where

$$f_o = f(o) \tag{3.59}$$

and

$$f_1 = \frac{W_{K+2} - W_{K+1}}{\Delta t} \tag{3.60}$$

Substituting (3.53) and (3.54) into (3.60) one gets

$$f_1 = \frac{W_K}{\Delta t} \varepsilon(\varepsilon-1) \tag{3.61}$$

Similarly

$$f_2 = \frac{W_K}{\Delta t} \varepsilon^2(\varepsilon-1) \tag{3.62}$$

where

$$f_m = f(m.\Delta t) \tag{3.63}$$

further

$$\dot{f}(t) = \frac{d\ f\ (t)}{dt} \tag{3.64}$$

in a discrete form

$$\dot{f}_o = \frac{f_1 - f_o}{\Delta t} \tag{3.65}$$

hence using (3.58) and (3.61)

$$\dot{f}_o = \frac{W_K}{(\Delta t)^2} \ (\varepsilon - 1)^2 \tag{3.66}$$

and

$$\dot{f}_1 = \frac{W_K}{(\Delta t)^2} \ \varepsilon(\varepsilon - 1)^2 \tag{3.67}$$

where

$$\dot{f}_m = \dot{f}(t) \Big|_{(m.\Delta t)} \tag{3.68}$$

since

$$\ddot{f}\ (t) = \frac{d\ \dot{f}\ (t)}{dt} \tag{3.69}$$

in a discrete form

$$\ddot{f}_o = \frac{\dot{f}_1 - \dot{f}_o}{\Delta t} \tag{3.70}$$

hence, using (3.66) and (3.67) one gets

$$\ddot{f}_o = \frac{W_K}{(\Delta t)^3} \ (\varepsilon - 1)^3 \tag{3.71}$$

d - proper number of sections

The results obtained in the last two sections will now be applied to obtain an estimate of the minimum number of sections to which the line should be divided in order to meet a specified accuracy limit.

Let the total allowable error be ΔW_t defined as :

$$\Delta W_t = n \cdot \varepsilon_a \tag{3.72}$$

Denote the relative error in calculating W by μ so that

$$\mu = \frac{\Delta W_t}{W_K} \tag{3.73}$$

Also we have by (3.52)

$$\varepsilon = \exp[- \frac{\alpha}{n}] \tag{3.74}$$

where

$$\alpha = \tfrac{1}{2}[\frac{R}{\rho} + G \rho] \tag{3.75}$$

In the practical case where $r \ll \rho$ and $g \ll \frac{1}{\rho}$, $\frac{\alpha}{n} \ll 1$ and ε can be approximated by

$$\varepsilon = 1-\frac{\alpha}{n} \tag{3.76}$$

Using this expression for ε we obtain the following results.

A - First formulation

Substituting (3.66) into (3.43) one obtains

$$(1-\varepsilon) < (2\varepsilon_a/W_K)^{\frac{1}{2}} \tag{3.77}$$

further substitute (3.72) and (3.73) into (3.77) to obtain

$$(1-\varepsilon) < \sqrt{\frac{2\mu}{n}} \tag{3.78}$$

Using (3.76) in (3.78) one obtains

$$\frac{\alpha}{n} < \sqrt{\frac{2\mu}{n}} \tag{3.79}$$

or

$$n > \frac{\alpha^2}{2\mu} \tag{3.80}$$

Thus the minimum number of sections to be chosen for the first formulation is given by

$$n_1 = \frac{\alpha^2}{2\mu} \tag{3.81}$$

B - Second formulation

Substituting (3.71) into (3.49) one obtains

$$1-\varepsilon < \left(\frac{12\varepsilon_a}{W_K} \right)^{1/3} \tag{3.82}$$

We obtain upon substitution of (3.72) and (3.73) into (3.82) we get

$$1-\varepsilon < \left(\frac{12\mu}{n} \right)^{1/3} \tag{3.83}$$

Then (3.76) and (3.83) yield

$$\frac{\alpha}{n} < \left(\frac{12\mu}{n} \right)^{1/3} \tag{3.84}$$

or

$$n > [\alpha^3/12\mu]^{\frac{1}{2}} \tag{3.85}$$

Thus the minimum number of sections to be chosen for the second formulation is given by

$$n_2 = \left(\frac{\alpha^3}{12\mu} \right)^{\frac{1}{2}} \tag{3.86}$$

3.6 Comparison between the two formulations

In this section it is shown that for most practical transmission lines to achieve the same accuracy by using the two given formulations, the first formulation requires a larger number of sections than that required by the second formulation.

Let

$$p = n_1^2 - n_2^2 \tag{3.87}$$

where n_1 and n_2 are the minimum number of sections for a given accuracy measure μ required for the first and second formulations respectively as given by (3.81) and (3.86).

Hence

$$p = \frac{\alpha^3}{4\mu} \left[\frac{\alpha}{\mu} - \frac{1}{3} \right] \tag{3.88}$$

The required relative error normally does not exceed 0.01, thus we can assume

$$\mu \lesssim 0.01 \tag{3.89}$$

Now by definition

$$\alpha = \tfrac{1}{2}\left[\frac{R}{\rho} + G\rho \right] \tag{3.90}$$

so that using (3.89) and (3.90)

If

$$\frac{R}{\rho} + G\,\rho > \frac{1}{150} \tag{3.91}$$

then

$$p > 0 \text{ and } n_1 > n_2$$

table 3.1 gives a comparison between n_1 and n_2 for different values of α.

α	n_1	n_2
0.99999964E-01	0.49999967E-01	0.28867505E-01
0.19999993E 00	0.19999987E 00	0.81649601E-01
0.29999989E 00	0.44999969E 00	0.14999998E 00
0.39999986E 00	0.79999948E 00	0.23094004E 00
0.49999982E 00	0.12499990E 01	0.32274854E 00
0.59999979E 00	0.17999983E 01	0.42426401E 00
0.69999975E 00	0.24499979E 01	0.53463376E 00
0.79999971E 00	0.31999979E 01	0.65319717E 00
0.89999968E 00	0.40499983E 01	0.77942276E 00
0.99999964E 00	0.49999981E 01	0.91287082E 00

TABLE 3.1 VARIATION OF n_1 AND n_2 WITH α FOR μ=0.1

CHAPTER 4

TRANSIENT ANALYSIS OF LOSSY NONUNIFORM LINES

4.1 Dividing the line

Consider a lossy transmission line characterized by distributed inductance $L(Z) > 0$, capacitance $C(Z) > 0$, resistance $R(Z) > 0$ and conductance $G(Z) > 0$ per unit length, where Z is the physical position on the line. In this case the relations given by equations (3.1) and (3.2) hold true, but with the transmission line parameters varying with Z. This means that the characteristic curves in the Z-t plane are no longer straight lines as is the case for uniform lines.

The electrical position along the line is defined by

$$Y(Z) = \int_0^Z \sqrt{L(\xi)\, C(\xi)}\; d\xi \tag{4.1}$$

From this definition it is evident that the characteristic curves in the Y-t plane are straight lines. It is worth noting here that the electrical position is the time delay that a wave initiated at Z=0 takes to arrive at the physical position Z.

Thus if we have a line whose length is h, the total time delay would be

$$\zeta = \int_0^h \sqrt{L(\xi)\, C(\xi)}\; d\xi \tag{4.2}$$

Now if the physical line length was divided into any number of equal sections, the electrical length (delay) of these sections will

not be equal. Hence we need to divide the line into n sections of equal electrical length, the method that will be used is as follows:

I - Divide the physical line length h into N sections, so that each section is of physical length $\frac{h}{N}$.

Thus we have created an N-component vector $[Z_K]$ representing the physical length between the sending end and the $(K+1)^{st}$ node.

With $Z_K = (K - 1).\frac{h}{N} \qquad K = 2,\ldots, N+1$ (4.3)

II - Using (4.1), another N-component vector $[Y_K]$ representing the electrical length between the sending end and the (K+1)st node corresponding to the above divisions.

III - Now the electrical line length ζ is divided into n equal sections. The delay of each is Δt.

$$\Delta t = \frac{\zeta}{n} \tag{4.4}$$

Thus a new n-component vector $[Y_j]$ is created

$$Y_j = j * \Delta t \qquad j = 1,\ldots., n \tag{4.5}$$

IV - The physical position vector $[Z_j]$ corresponding to the vector $[Y_j]$ is obtained using the linear interpolation formula

$$Z_{j+1} = Z_K + \frac{(Z_{K+1} - Z_K)\ (Y_j - Y_K)}{(Y_{K+1} - Y_K)} \qquad j=1,\ldots, n-1 \tag{4.6}$$

or

$$Z_{j+1} = Z_K + \frac{h\ (Y_j - Y_K)}{N\ (Y_{K+1} - Y_K)} \tag{4.7}$$

Thus the line is now divided into n sections of equal delay.

4.2 The main equations

Rewrite equation (3.1)

$$\frac{d}{dt}\left[V(Z,t) \pm \sqrt{\frac{L(Z)}{C(Z)}}\ i(Z,t)\right] =$$

$$-\left[\frac{G(Z)}{C(Z)}\ V(Z,t) \pm \frac{R(Z)}{\sqrt{L(Z)\ C(Z)}}\ i\ (Z,t)\right] \tag{4.8}$$

Let

$$W(Z,t) = V(Z,t) + \sqrt{\frac{L(Z)}{C(Z)}}\ i\ (Z,t) \tag{4.9}$$

$$f(Z,t) = -\left[\frac{G(Z)}{C(Z)}\ V\ (Z,t) + \frac{R(Z)}{\sqrt{L(Z)\ C(Z)}}\ i\ (Z,t)\right] \tag{4.10}$$

Thus one of the two equations given by (4.8) is transformed to

$$\frac{d\ W(Z,t)}{dt} = f(Z,t) \tag{4.11}$$

Consider the time interval $[t - T, t]$ with $T = \Delta t$ being the time delay of each section as given by (4.4), then equation (4.11) can be written in a discrete form as

$$\Delta W = f_{avg}(Z,t) \cdot \Delta t \tag{4.12}$$

Now, if we consider that at time t the wave W is at the K^{th} node, then

$$\Delta W = W_K(t) - W_{K-1}(t - T) \tag{4.13}$$

In section (3.6) it was shown that the second formulation is superior to the first formulation, so we adopt the former in approximating $f(Z,t)\,\big|_{avg}$ so that we take

$$f(Z,t)\,\big|_{avg} = \frac{1}{2}\,[f_{K-1}(t - T) + f_K(t)] \tag{4.14}$$

Thus we have for the forward propagating argument

$$V_K(t) + \rho_K\, i_K(t) - V_{K-1}(t - T) - \rho_{K-1}\, i_{K-1}(t - T) =$$

$$- \frac{\Delta t}{2}\,[\,\frac{G_K}{C_K}\, V_K(t) + \frac{R_K}{\sqrt{L_K C_K}}\, i_K(t) +$$

$$\frac{G_{K-1}}{C_{K-1}}\, V_{K-1}(t - T) + \frac{R_{K-1}}{\sqrt{L_{K-1}\, C_{K-1}}}\, i_{K-1}(t - T)] \tag{4.15}$$

or

$$V_K(t)\,[1 + \frac{\Delta t}{2}\,\frac{G_K}{C_K}] + [\rho_K + \frac{\Delta t}{2}\,\frac{R_K}{\sqrt{L_K C_K}}\,]\, i_K(t) =$$

$$V_{K-1}(t - T)\,[1 - \frac{\Delta t}{2}\,\frac{G_{K-1}}{C_{K-1}}] + [\rho_{K-1} - \frac{\Delta t}{2}\,\frac{R_{K-1}}{\sqrt{L_{K-1}\, C_{k-1}}}]\, i_{K-1}(t - T)$$

$$K=2,\ldots,N \tag{4.16}$$

Similarly for the backward propagating wave

$$V_{K-1}(t)\ [1 + \frac{\Delta t}{2}\ \frac{G_{K-1}}{C_{K-1}}] - [\rho_{K-1} + \frac{\Delta t}{2}\ \frac{R_{K-1}}{\sqrt{L_{K-1}\ C_{K-1}}}]\ i_{K-1}(t) =$$

$$V_K(t - T)\ [1 - \frac{\Delta t}{2}\ \frac{G_K}{C_K}] - [\rho_K - \frac{\Delta t}{2}\ \frac{R_K}{\sqrt{L_K C_K}}] i_K(t - T)$$

$$K=2,\ldots N \qquad (4.17)$$

Let

$$a_K = \Delta t\ \frac{R_K}{\sqrt{L_K C_K}} \qquad (4.18)$$

$$b_K = \Delta t\ \frac{G_K}{C_K} \qquad (4.19)$$

Then (4.16) and (4.17) transform to

$$V_K(t)\ [1 + \frac{b_K}{2}\ ] + [\rho_K + \frac{a_K}{2}\ ]\ i_K\ (t) =$$

$$V_{K-1}(t - T)\ [1 - \frac{b_{K-1}}{2}\ ] + [\rho_{K-1} - \frac{a_{K-1}}{2}\ ]\ i_{K-1}(t - T) \qquad (4.20)$$

$$V_{K-1}(t)\ [1 + \frac{b_{K-1}}{2}\ ] - [\rho_{K-1} + \frac{a_{K-1}}{2}\ ]\ i_{K-1}(t) =$$

$$V_K\ (t - T)\ [1 - \frac{b_K}{2}\ ] - [\rho_K - \frac{a_K}{2}\ ]\ i_K(t - T) \qquad (4.21)$$

Rewrite (4.21) between the K^{th} and $(K + 1)^{st}$ nodes, we have

$$V_K(t)\ [1 + \frac{b_K}{2}\ ] - [\rho_K + \frac{a_K}{2}\ ]\ i_K(t) =$$

$$V_{K+1}(t - T) \left[1 - \frac{b_{K+1}}{2}\right] - \left[\rho_{K+1} - \frac{a_{K+1}}{2}\right] i_{K+1}(t - T) \quad (4.22)$$

further adding (4.20) to (4.22) and subtracting (4.22) from (4.20) we get

$$V_K(t) = \Big[\left(1 - \frac{b_{K-1}}{2}\right) V_{K-1}(t - T) +$$

$$\left(1 - \frac{b_{K+1}}{2}\right) V_{K+1}(t - T) + \left(\rho_{K-1} - \frac{a_{K-1}}{2}\right) i_{K-1}(t - T) -$$

$$\left(\rho_{K+1} - \frac{a_{K+1}}{2}\right) i_{K+1}(t - T)\Big] / [2 + b_K] \qquad (4.23)$$

$$i_K(t) = \Big[\left(1 - \frac{b_{K-1}}{2}\right) V_{K-1}(t - T) -$$

$$\left(1 - \frac{b_{K+1}}{2}\right) V_{K+1}(t - T) + \left(\rho_{K-1} - \frac{a_{K-1}}{2}\right) i_{K-1}(t - T)$$

$$+ \left(\rho_{K+1} - \frac{a_{K+1}}{2}\right) i_{K+1}(t - T)\Big]/[2\rho_K + a_K] \qquad (4.24)$$

CHAPTER 5

DESCRIPTION OF COMPUTER PROGRAM

The algorithms discussed in chapters 3 and 4 were incorporated into a Fortran IV computer program. The operation and flow of the main program are described in this chapter.

A sample computer output listing is given to illustrate the operation of the program.

5.1 Computer program flow chart

The flow of the program is shown in fig. (5-1) and the program listing is given in Appendix C.

The program reads in data cards, describing the system parameters, accuracy limits and type of transmission line whose transient response is to be analyzed. The program then writes out all input parameters, type of transmission line (uniform or non-uniform) and maximum permissible relative error (in case of uniform lines).

In the case when a uniform line is analyzed, the program determines the appropriate number of sections n. If the computed value of n is less than 10, the program sets n to be equal to 10 and the main loop is entered. Also if the computed value of n is less than 1000 the main loop is entered. If the computed value of n is larger than 1000, the program writes out a message indicating that the number of sections has exceeded the maximum permissible value and the program stops. It should be noted that this limit

can be increased if desired by a minor change in the program at the expense of higher storage requirements.

If the line is nonuniform, the calculation of n is skipped and the main loop is entered. Here n is given in the data.

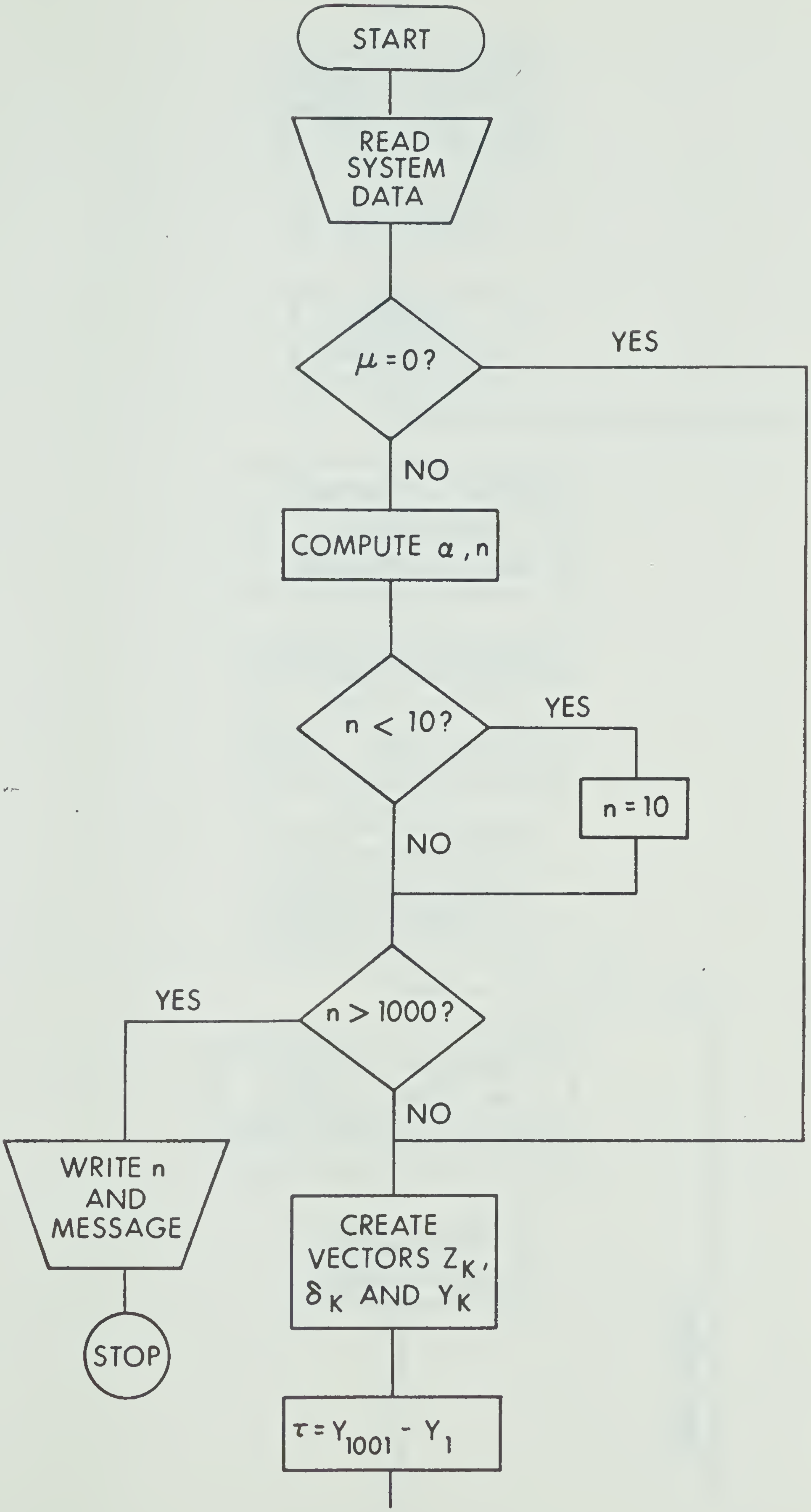
The first step in the main loop is to divide the line's physical length into one thousand sections of equal length. It is assumed that the line is of unit length. The vector Z_K of physical length between the sending end and the K^{th} node is created. Then the corresponding electrical length vector Y_K is computed using a trapezoidal rule formulation.

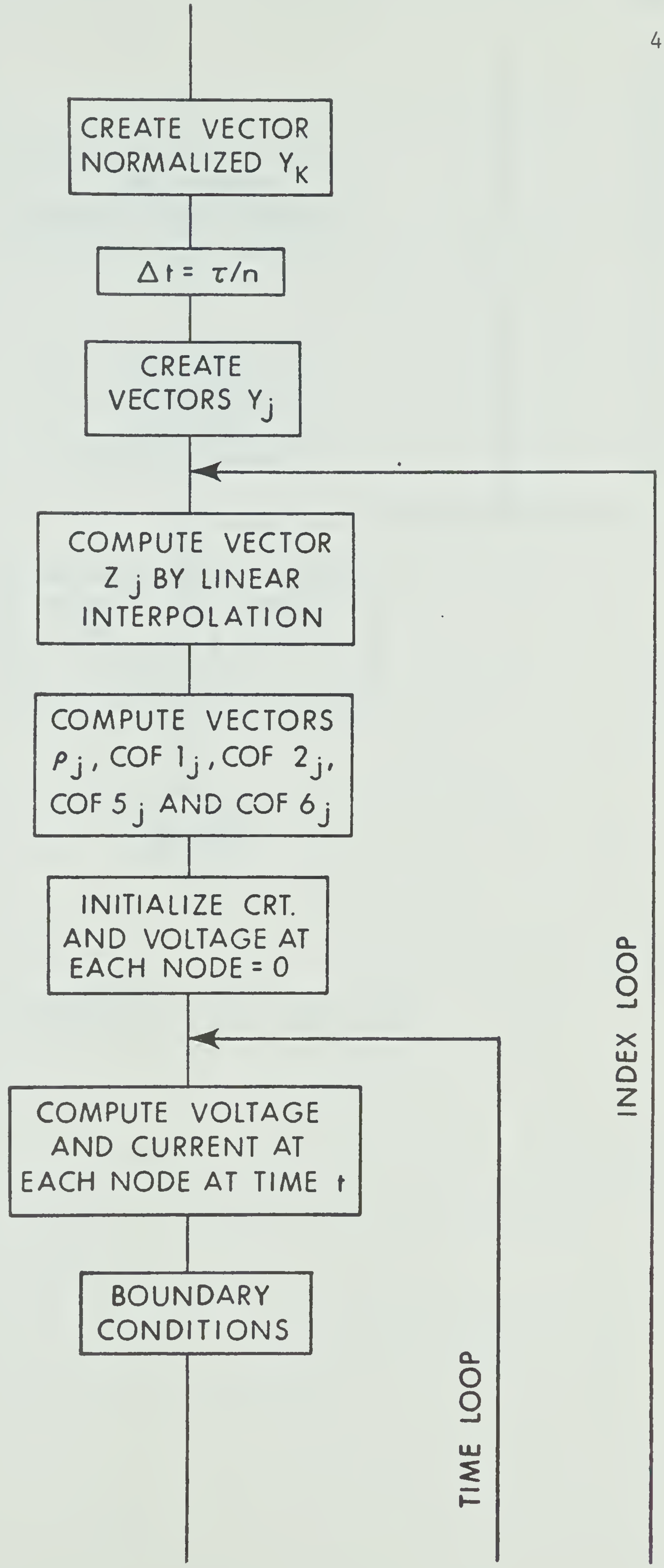
The line's electrical length ζ is then computed and the vector of normalized electrical length is created $Y_K^{(n)}$ by dividing Y_K through ζ.

The program then computes the appropriate time step Δt as the out-come of dividing the total electrical length by the number of sections n. The line is then divided into n sections of equal electrical length (delay) by creating the vector Y_j according to the rule $Y_j = \Delta t.j$.

Further the physical length vector Z_j corresponding to Y_j is computed using a linear interpolation formula. With the vector Z_j created, the vectors ρ_j, Cof_{1j}, Cof_{2j}, Cof_{5j} and Cof_{6j} are computed as a prior step to applying the current and voltage relations.

The initial values of current and voltage at each node are then stored as zeros in the C and V arrays.


START
READ SYSTEM DATA
μ = 0?
YES
NO
COMPUTE α , n
n < 10?
YES
n = 10
NO
n > 1000?
YES
NO
WRITE n AND MESSAGE
STOP
CREATE VECTORS Z_K, δ_K AND Y_K
$\tau = Y_{1001} - Y_1$



CREATE VECTOR NORMALIZED Y_K
$\Delta t = \tau/n$
CREATE VECTORS Y_j
COMPUTE VECTOR Z_j BY LINEAR INTERPOLATION
COMPUTE VECTORS ρ_j, COF 1_j, COF 2_j, COF 5_j AND COF 6_j
INITIALIZE CRT. AND VOLTAGE AT EACH NODE = 0
COMPUTE VOLTAGE AND CURRENT AT EACH NODE AT TIME t
BOUNDARY CONDITIONS
TIME LOOP
INDEX LOOP

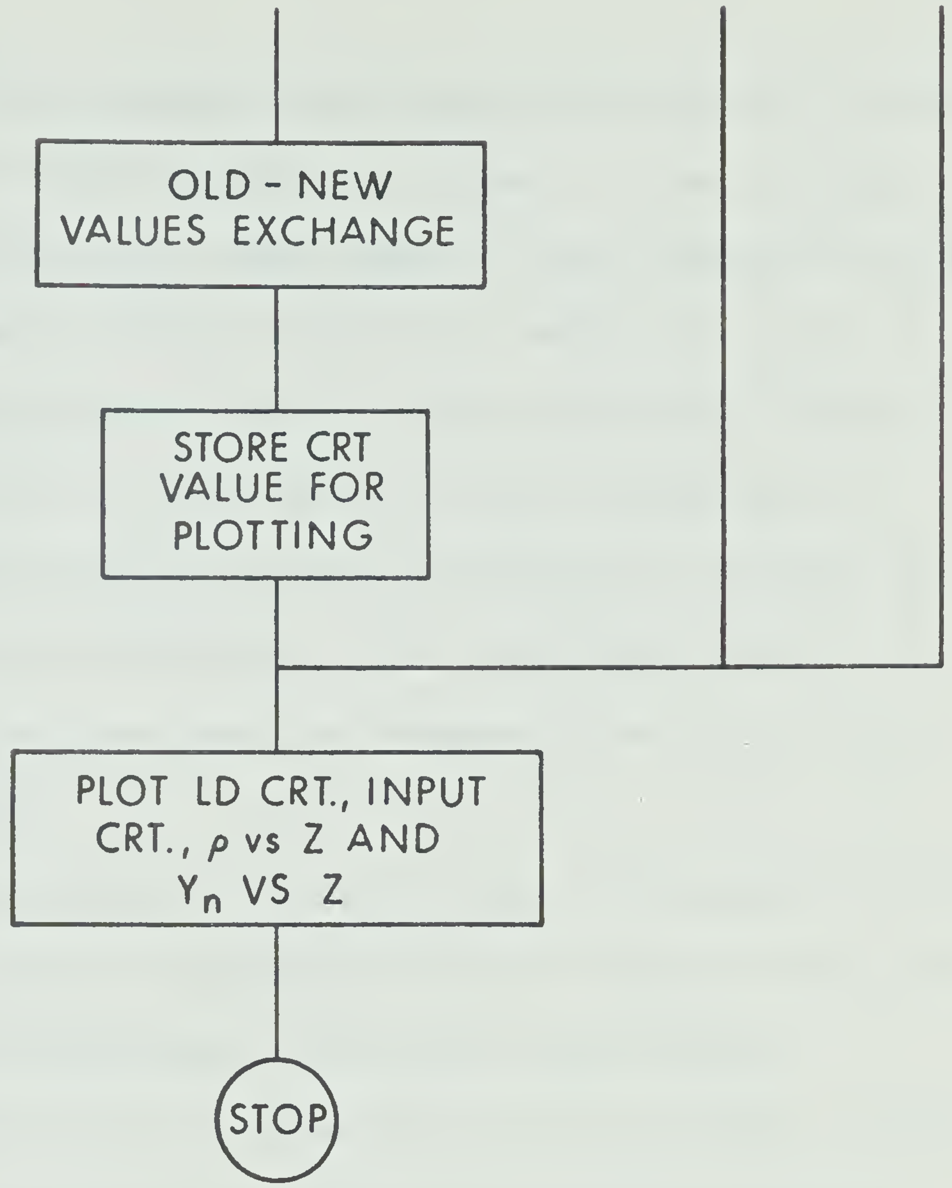


FIG. 5.1 COMPUTER PROGRAM FLOW CHART

In this program C and V denote old values of current and voltage, while CC and VV denote new values of these variables. By old and new values, it is meant values at time instants $(t-n.\Delta t)$ and at t respectively. The next step is to compute the values of new currents and voltages at each node using old values. Then the program replaces the old values by new values to determine the currents and voltages at the next time instant and so on until the required time interval is finished. Thus the program destroys old values of voltages at every node and currents at every node except at the receiving end.

The program then writes out the values of currents at the sending end, one quarter line length, half line length, three quarter line length and receiving end at every fifth time instant.

The last step in the program is to plot the input current wave, load current wave versus time. Also the program will plot the variation of characteristic impedance ρ and normalized electrical length with the physical length.

5-2 User supplied data

The computer program user must supply data to the program describing the characteristics of the system, the calculation accuracy or number of sections.

These data are supplied to the program by means of data cards at the end of the Fortran Card deck.

The parameters and the formats on each data card are described in this section.

The first data card contains the load resistance RR and the source shunt conductance GG. The format is 2E16.8.

The second data card contains XNN and XMEW. These are the number of sections n and the relative error allowed μ. If the value of μ supplied is zero, the program takes the line as a non-uniform one. However, in this case the value of n must be entered. The format is 2E16.8.

The third data card contains RESI, RØVAL and GVAL, these values are to be entered if the line is uniform. RESI is the value of the resistance per unit length, RØVAL is the value of the characteristic impedence and GVAL is the value of the shunt conductance of the line per unit length. The format is 3E16.8.

The fourth data card contains XLO, CO, RO, GO. These are the amplitudes of inductance, capacitance, resistance and conductance per unit length functions describing the variation of these parameters with the physical length. The format is 4E16.8.

The fifth data card contains XLE1, XLE2, XLE3, AMP and SLOPE. These parameters describe the source current characteristic as a trapezoidal wave. This wave is shown in fig. 5-2 with the meaning of each of the above mentioned parameters.
It is noted that SLOPE = AMP/XLE1. The format is 5E12.4.

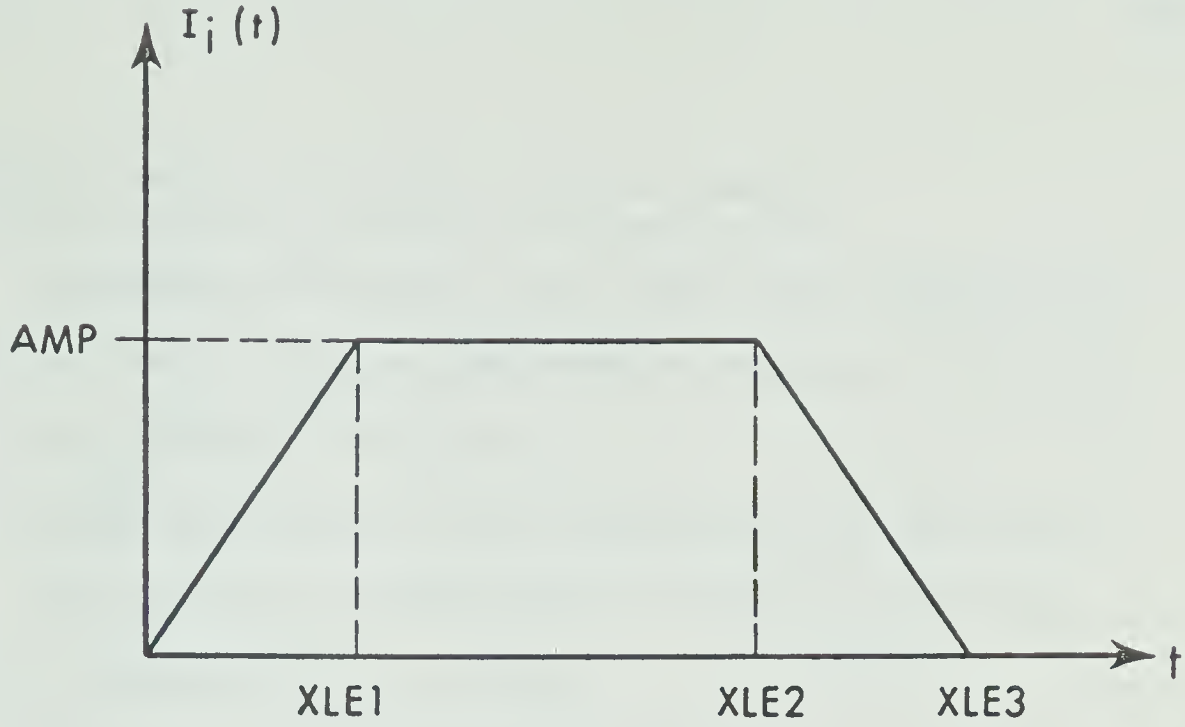


FIG. 5.2 TRAPEZOIDAL WAVE FORM

In addition to the above mentioned five data cards the following functions should be supplied.

The first function is FL(X), which describes the variation of the line's inductance with the length X. The second function is FC(X) for the capacitance. The third function is FR(X) for the resistance. The fourth function is FG(X) for the shunt conductance, and the fifth function is the source current function SOUR(X).

As an example the following functions were assumed for the above mentioned variations.

FL(X) = XLO* (2.0 + SIN(πX))

FC(X) = CO * EXP (-X)

FR(X) = RO * Log (1 + X)

FG(X) = GO

5-3 Numerical study of the effect of n on results

The program described in this chapter was equipped with a facility to compute and plot the transient response of the given line for three different values of n.

This is facilitated by the introduction of a three step DØ loop. The first for the given value of n say n_o, the second for $n = 5n_o$ and the third for $n = 20n_o$.

Another trial was for n_o, $1.5n_o$ and $2n_o$ respectively.

The computed values of currents at the locations on the line previously mentioned and plots of load current versus time are shown in figs. (5-3) and (5-4).

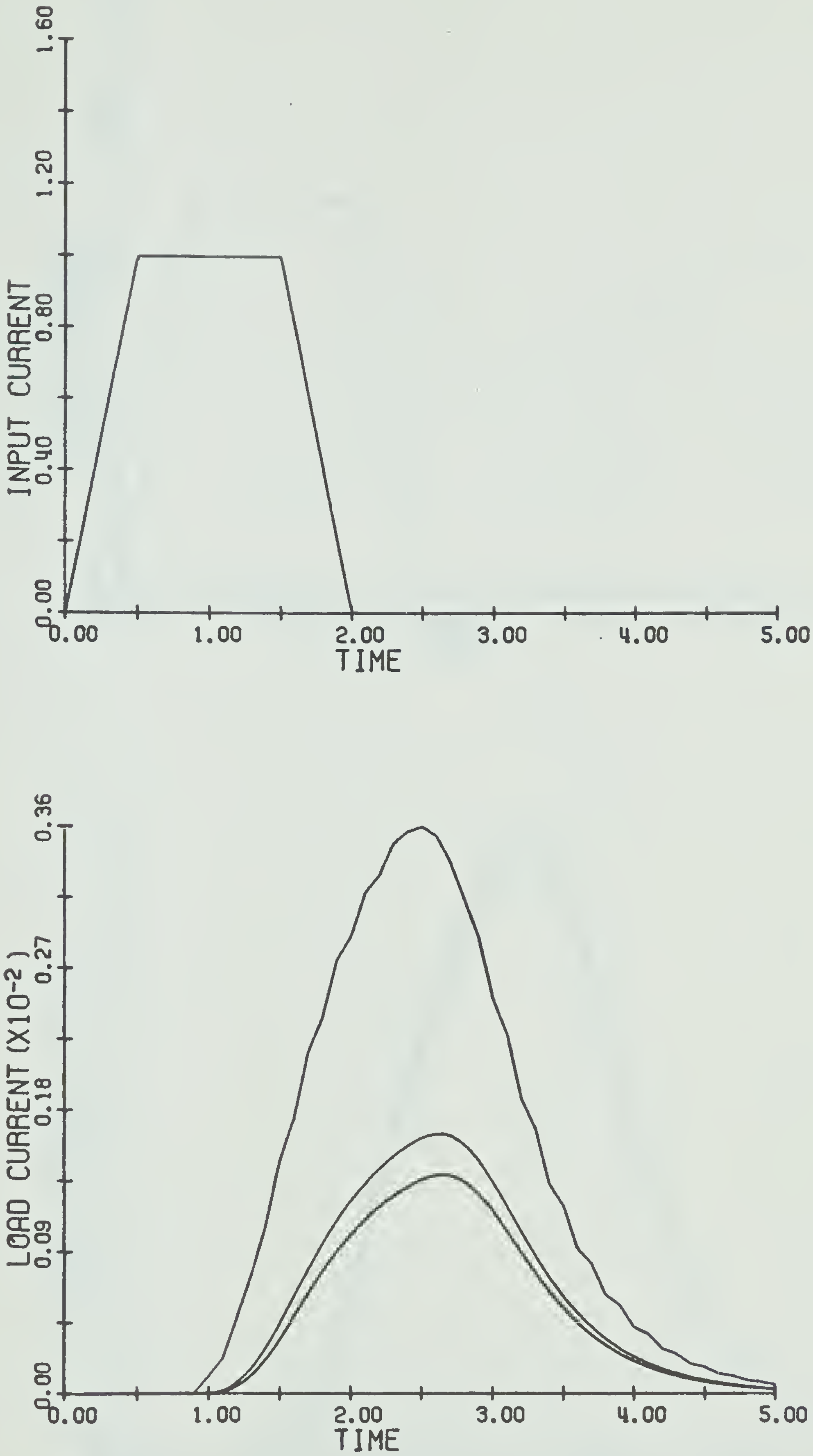


FIG. 5.3 TRANSIENT RESPONSE FOR

n=10, 50 AND 200

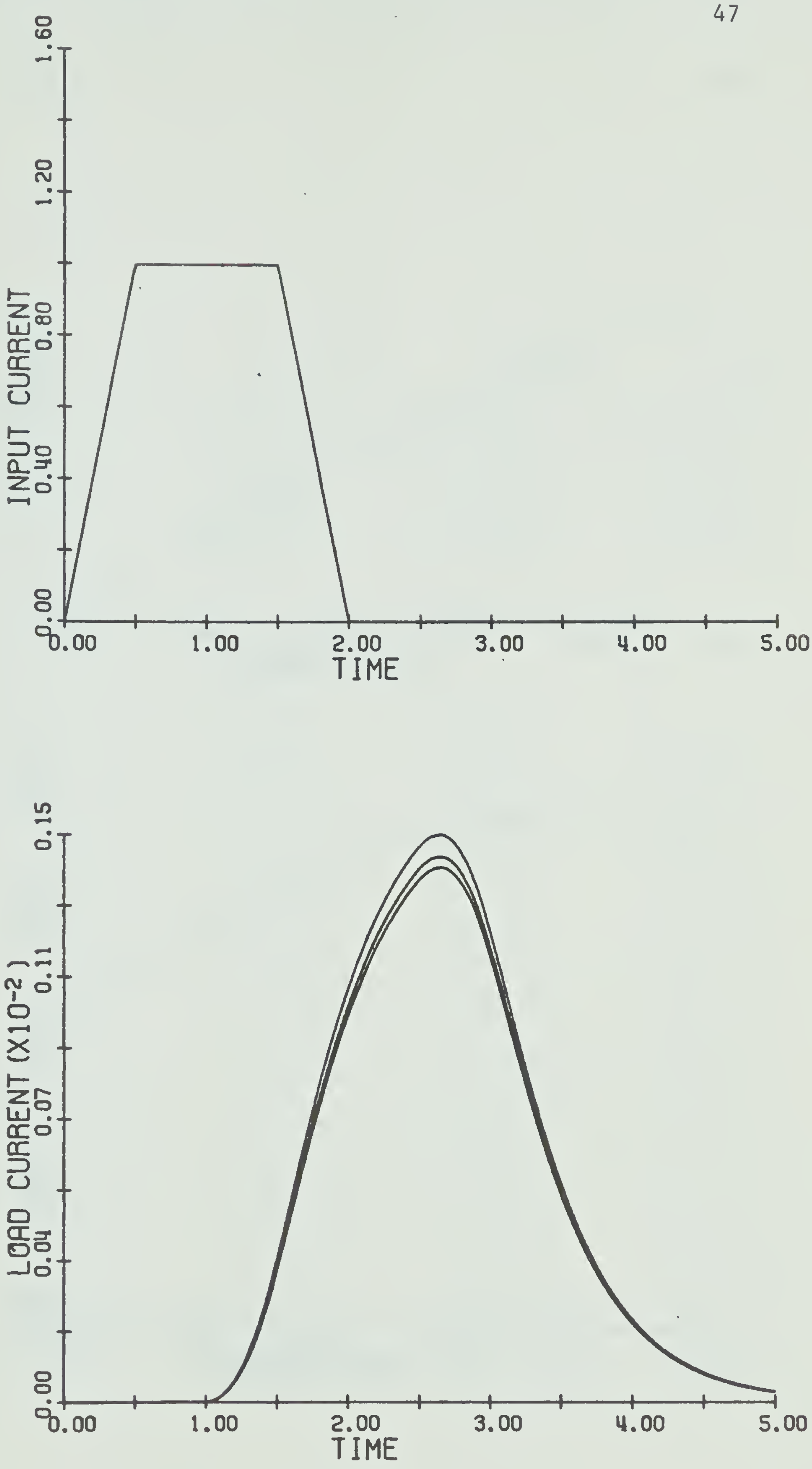
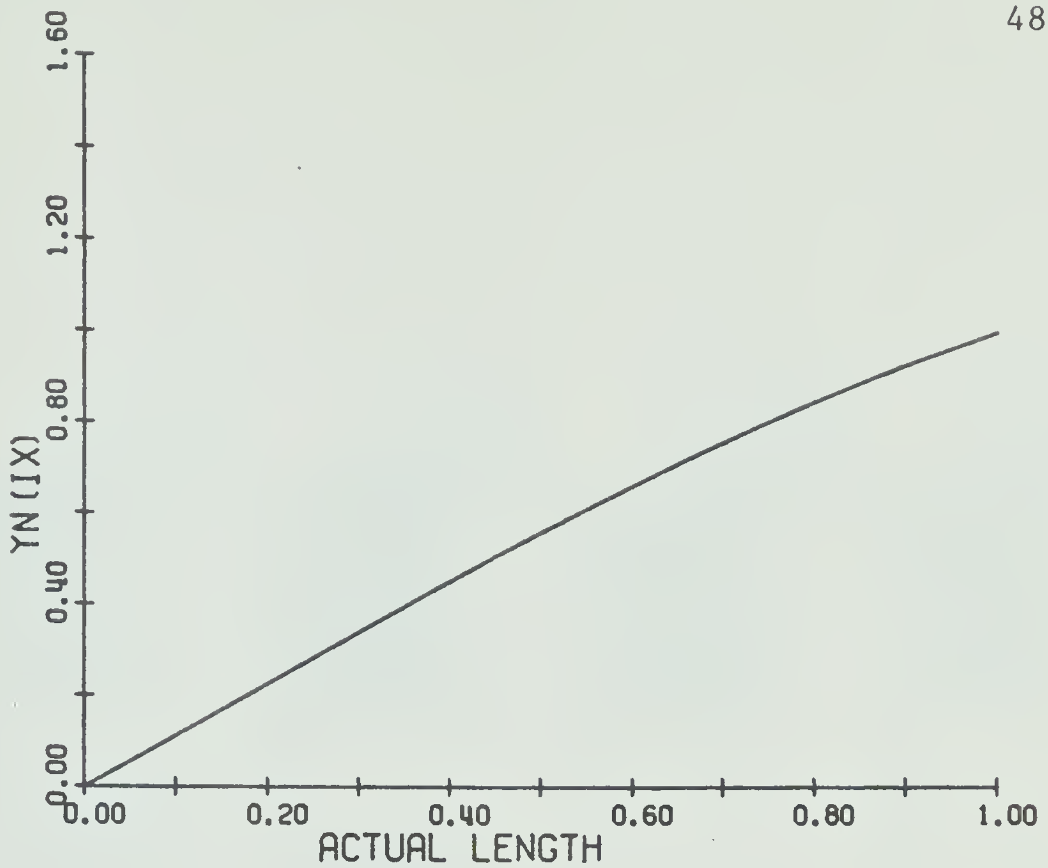
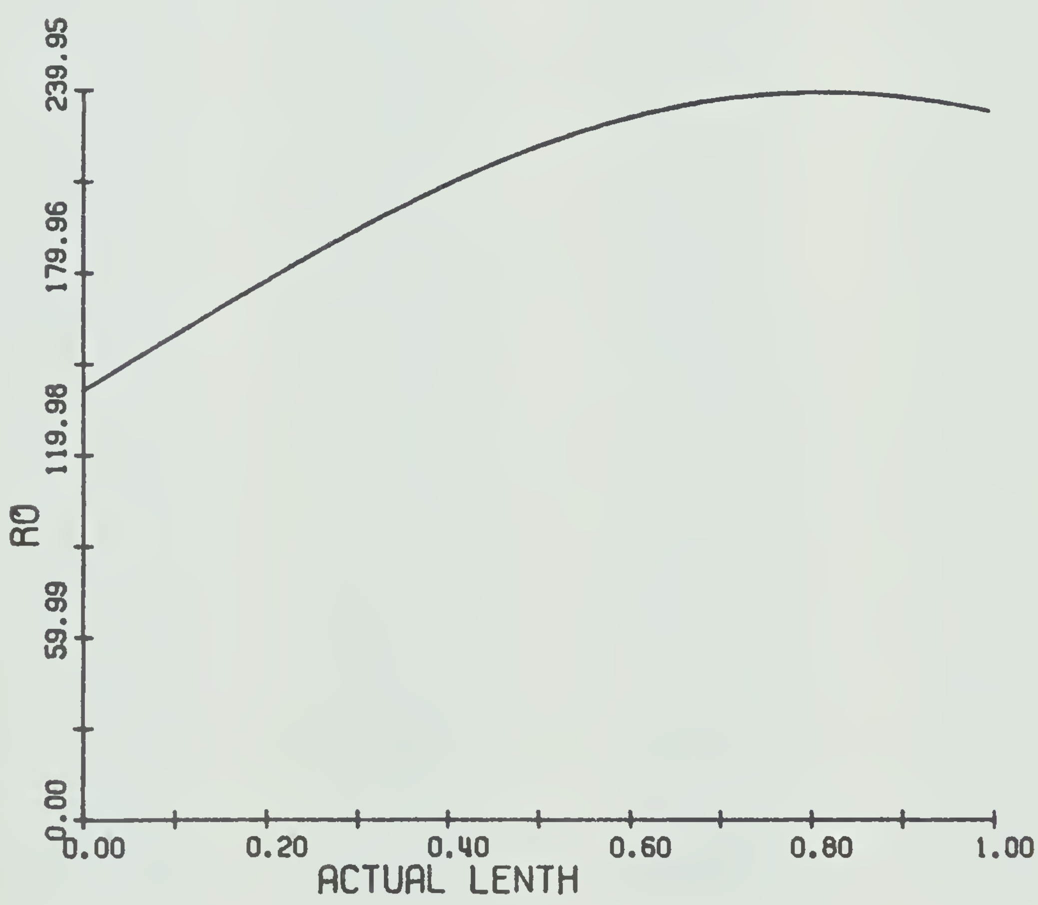


FIG. 5.4 TRANSIENT RESPONSE FOR

n=100, 150 AND 200


1.60
1.20
0.80
0.40
0.00
YN(IX)
0.00
0.20
0.40
0.60
0.80
1.00
ACTUAL LENGTH


239.95
179.96
119.98
59.99
0.00
RO
0.00
0.20
0.40
0.60
0.80
1.00
ACTUAL LENTH

0.8000000E 00	0.4569398E 00	0.1179693E-01	0.0	0.0
0.1000000E 01	0.6435866E 00	0.9511560E-01	0.1199614E-01	0.1093948E-03
0.1000000E 01	0.6685686E 00	0.1312146E 00	0.2925690E-01	0.1458441E-02
0.2000008E 00	0.2170602E 00	0.1331436E 00	0.3801805E-01	0.2866412E-02
0.0	0.3248560E-01	0.5361597E-01	0.2876412E-01	0.3550411E-02
0.0	0.8078363E-02	0.1906353E-01	0.1266391E-01	0.2467589E-02
0.0	0.2882114E-02	0.5782910E-02	0.4243717E-02	0.1174732E-02
0.0	0.8772933E-03	0.2175869E-02	0.1643409E-02	0.4155547E-03
0.0	0.3304961E-03	0.6828641E-03	0.5241956E-03	0.1636188E-03
0.0	0.1037963E-03	0.2597007E-03	0.2003469E-03	0.5267253E-04
0.1999999E 00	0.0	0.0	0.0	0.0
0.4000000E 00	0.0	0.0	0.0	0.0
0.5999999E 00	0.3080618E-01	0.0	0.0	0.0
0.8000000E 00	0.7996100E-01	0.0	0.0	0.0
0.9599999E 00	0.1365302E 00	0.6195493E-03	0.0	0.0
0.1000000E 01	0.1985610E 00	0.5232073E-02	0.0	0.0
0.1000000E 01	0.2641563E 00	0.1269520E-01	0.0	0.0
0.1000000E 01	0.3008220E 00	0.2226996E-01	0.2227691E-03	0.0
0.1000000E 01	0.3229754E 00	0.3367817E-01	0.8659367E-03	0.0
0.1000000E 01	0.3379491E 00	0.4582062E-01	0.1894894E-02	0.1514323E-05
0.1000000E 01	0.3494176E 00	0.5492588E-01	0.3316215E-02	0.2057997E-04
0.1000000E 01	0.3576394E 00	0.6248513E-01	0.5062412E-02	0.7116888E-04
0.1000000E 01	0.3641481E 00	0.6832713E-01	0.6860722E-02	0.1561420E-03
0.1000000E 01	0.3689565E 00	0.7317370E-01	0.8544430E-02	0.2795646E-03
0.1000000E 01	0.3728333E 00	0.7692069E-01	0.1000185E-01	0.4355770E-03
0.8000011E 00	0.3757478E 00	0.8003211E-01	0.1129845E-01	0.6054251E-03
0.6000004E 00	0.3781241E 00	0.8244067E-01	0.1237013E-01	0.7764511E-03
0.4000015E 00	0.3491244E 00	0.8444303E-01	0.1329944E-01	0.9337922E-03
0.2000008E 00	0.3014531E 00	0.8599538E-01	0.1405025E-01	0.1079261E-02
0.4000092E-01	0.2460197E 00	0.8666772E-01	0.1469234E-01	0.1203923E-02
0.0	0.1849270E 00	0.8305788E-01	0.1520436E-01	0.1314523E-02
0.0	0.1200532E 00	0.7643008E-01	0.1563859E-01	0.1405859E-02
0.0	0.8398533E-01	0.6750405E-01	0.1575930E-01	0.1485075E-02
0.0	0.6229385E-01	0.5663674E-01	0.1540590E-01	0.1549115E-02
0.0	0.4770354E-01	0.4491479E-01	0.1460493E-01	0.1602399E-02
0.0	0.3653200E-01	0.3616019E-01	0.1337530E-01	0.1627061E-02
0.0	0.2855720E-01	0.2887370E-01	0.1177942E-01	0.1613578E-02
0.0	0.2223997E-01	0.2325925E-01	0.1010722E-01	0.1557969E-02
0.0	0.1759101E-01	0.1858965E-01	0.8522157E-02	0.1459330E-02
0.0	0.1383808E-01	0.1499033E-01	0.7147375E-02	0.1322827E-02
0.0	0.1102665E-01	0.1199392E-01	0.5915325E-02	0.1169388E-02
0.0	0.8730456E-02	0.9681161E-02	0.4897650E-02	0.1011231E-02
0.0	0.6990764E-02	0.7753406E-02	0.4010502E-02	0.8646892E-03
0.0	0.5558982E-02	0.6263349E-02	0.3294913E-02	0.7276691E-03
0.0	0.4466049E-02	0.5019918E-02	0.2680305E-02	0.6100861E-03
0.0	0.3561905E-02	0.4057627E-02	0.2191249E-02	0.5050148E-03
0.0	0.2868184E-02	0.3253882E-02	0.1774923E-02	0.4183054E-03
0.0	0.2292232E-02	0.2631263E-02	0.1446396E-02	0.3427006E-03
0.0	0.1848745E-02	0.2110868E-02	0.1168286E-02	0.2816790E-03
0.0	0.1479618E-02	0.1707478E-02	0.9500002E-03	0.2292354E-03
0.0	0.1194675E-02	0.1370152E-02	0.7658843E-03	0.1874745E-03
0.0	0.9570925E-03	0.1108545E-02	0.6218820E-03	0.1519041E-03
0.0	0.7733721E-03	0.8897078E-03	0.5007146E-03	0.1238189E-03
0.0	0.6200001E-03	0.7199359E-03	0.4061693E-03	0.1000340E-03
0.0	0.5012546E-03	0.5778880E-03	0.3267457E-03	0.8135765E-04

0.0	0.4020391E-03	0.4676627E-03	0.2648709E-03	0.6560038E-04
0.0	0.3251587E-03	0.3754226E-03	0.2129504E-03	0.5327264E-04
0.0	0.2608846E-03	0.3038356E-03	0.1725454E-03	0.4289765E-04
0.0	0.2110510E-03	0.2439232E-03	0.1386658E-03	0.3480061E-04
0.0	0.1693717E-03	0.1974214E-03	0.1123202E-03	0.2799767E-04
0.5000000E-01	0.0	0.0	0.0	0.0
0.9999996E-01	0.0	0.0	0.0	0.0
0.1500000E 00	0.0	0.0	0.0	0.0
0.1999999E 00	0.0	0.0	0.0	0.0
0.2500000E 00	0.0	0.0	0.0	0.0
0.3000000E 00	0.0	0.0	0.0	0.0
0.3499999E 00	0.0	0.0	0.0	0.0
0.4000000E 00	0.0	0.0	0.0	0.0
0.4499999E 00	0.0	0.0	0.0	0.0
0.5000000E 00	0.1321034E-02	0.0	0.0	0.0
0.5500000E 00	0.8377887E-02	0.0	0.0	0.0
0.5999999E 00	0.1633499E-01	0.0	0.0	0.0
0.6500000E 00	0.2502728E-01	0.0	0.0	0.0
0.6999999E 00	0.3442798E-01	0.0	0.0	0.0
0.7500000E 00	0.4441303E-01	0.0	0.0	0.0
0.8000000E 00	0.5496659E-01	0.0	0.0	0.0
0.8499999E 00	0.6599307E-01	0.0	0.0	0.0
0.9000000E 00	0.7748353E-01	0.0	0.0	0.0
0.9499999E 00	0.8936238E-01	0.0	0.0	0.0
0.9999999E 00	0.1016248E 00	0.1083283E-03	0.0	0.0
0.1000000E 01	0.1142105E 00	0.7417507E-03	0.0	0.0
0.1000000E 01	0.1271170E 00	0.1567815E-02	0.0	0.0
0.1000000E 01	0.1402954E 00	0.2563274E-02	0.0	0.0
0.1000000E 01	0.1537443E 00	0.3733363E-02	0.0	0.0
0.1000000E 01	0.1674239E 00	0.5056720E-02	0.0	0.0
0.1000000E 01	0.1813340E 00	0.6537829E-02	0.0	0.0
0.1000000E 01	0.1954411E 00	0.8157190E-02	0.0	0.0
0.1000000E 01	0.2097450E 00	0.9918757E-02	0.0	0.0
0.1000000E 01	0.2242184E 00	0.1180477E-01	0.0	0.0
0.1000000E 01	0.2375407E 00	0.1381867E-01	0.5070567E-05	0.0
0.1000000E 01	0.2452039E 00	0.1594450E-01	0.3864350E-04	0.0
0.1000000E 01	0.2522522E 00	0.1818518E-01	0.9081286E-04	0.0
0.1000000E 01	0.2585679E 00	0.2052644E-01	0.1610400E-03	0.0
0.1000000E 01	0.2644120E 00	0.2297085E-01	0.2515251E-03	0.0
0.1000000E 01	0.2696837E 00	0.2550543E-01	0.3611504E-03	0.0
0.1000000E 01	0.2745857E 00	0.2813262E-01	0.4917916E-03	0.0
0.1000000E 01	0.2790315E 00	0.3084071E-01	0.6419457E-03	0.0
0.1000000E 01	0.2831826E 00	0.3363174E-01	0.8132237E-03	0.0
0.1000000E 01	0.2869637E 00	0.3649534E-01	0.1003881E-02	0.0
0.1000000E 01	0.2905061E 00	0.3932513E-01	0.1215307E-02	0.1607165E-06
0.1000000E 01	0.2937447E 00	0.4167993E-01	0.1445627E-02	0.1401719E-05
0.1000000E 01	0.2967870E 00	0.4393852E-01	0.1696036E-02	0.3724990E-05
0.1000000E 01	0.2995774E 00	0.4605766E-01	0.1964609E-02	0.7217894E-05
0.1000000E 01	0.3022044E 00	0.4808897E-01	0.2252384E-02	0.1213886E-04
0.1000000E 01	0.3046197E 00	0.4999368E-01	0.2557433E-02	0.1850646E-04
0.1000000E 01	0.3069980E 00	0.5181837E-01	0.2880663E-02	0.2655829E-04
0.1000000E 01	0.3089976E 00	0.5352866E-01	0.3220182E-02	0.3625485E-04
0.1000000E 01	0.3109815E 00	0.5516665E-01	0.3576786E-02	0.4781148E-04
0.1000000E 01	0.3128128E 00	0.5670145E-01	0.3948644E-02	0.6114223E-04
0.1000000E 01	0.3145461E 00	0.5817116E-01	0.4331373E-02	0.7644035E-04
0.1000000E 01	0.3161484E 00	0.5954764E-01	0.4698392E-02	0.9358450E-04
0.1000000E 01	0.3176667E 00	0.6086564E-01	0.5064506E-02	0.1127468E-03
0.1000000E 01	0.3190724E 00	0.6210001E-01	0.5422745E-02	0.1337800E-03
0.1000000E 01	0.3204058E 00	0.6328183E-01	0.5777583E-02	0.1568373E-03
0.1000000E 01	0.3216416E 00	0.6438833E-01	0.6122265E-02	0.1817536E-03

0.1000000E 01	0.3228149E 00	0.6544775E-01	0.6461833E-02	0.2086637E-03
0.1000000E 01	0.3239043E 00	0.6643981E-01	0.6789856E-02	0.2373929E-03
0.1000000E 01	0.3249389E 00	0.6738943E-01	0.7111639E-02	0.2680614E-03
0.1000000E 01	0.3259000E 00	0.6827867E-01	0.7421110E-02	0.3004870E-03
0.1000000E 01	0.3268140E 00	0.6913006E-01	0.7723689E-02	0.3346147E-03
0.9500008E 00	0.3276637E 00	0.6992698E-01	0.8013632E-02	0.3692608E-03
0.9000015E 00	0.3284723E 00	0.7068992E-01	0.8296363E-02	0.4047656E-03
0.8500004E 00	0.3292246E 00	0.7140434E-01	0.8566506E-02	0.4405011E-03
0.8000011E 00	0.3299409E 00	0.7208836E-01	0.8829344E-02	0.4767065E-03
0.7500000E 00	0.3306077E 00	0.7272875E-01	0.9079855E-02	0.5127268E-03
0.7000008E 00	0.3312430E 00	0.7334179E-01	0.9323120E-02	0.5488985E-03
0.6500015E 00	0.3318351E 00	0.7391602E-01	0.9554554E-02	0.5845579E-03
0.6000004E 00	0.3323989E 00	0.7446557E-01	0.9778913E-02	0.6201160E-03
0.5500011E 00	0.3329249E 00	0.7498032E-01	0.9992000E-02	0.6549151E-03
0.5000000E 00	0.3321052E 00	0.7547313E-01	0.1019832E-01	0.6894181E-03
0.4500008E 00	0.3255159E 00	0.7593453E-01	0.1039394E-01	0.7229832E-03
0.4000015E 00	0.3180048E 00	0.7637632E-01	0.1058311E-01	0.7561084E-03
0.3500004E 00	0.3097282E 00	0.7679015E-01	0.1076230E-01	0.7881739E-03
0.3000011E 00	0.3007243E 00	0.7718629E-01	0.1093538E-01	0.8196994E-03
0.2500000E 00	0.2911097E 00	0.7755738E-01	0.1109913E-01	0.8500898E-03
0.2000008E 00	0.2809093E 00	0.7791263E-01	0.1125716E-01	0.8798703E-03
0.1500015E 00	0.2702129E 00	0.7824528E-01	0.1140654E-01	0.9084775E-03
0.1000004E 00	0.2590374E 00	0.7856405E-01	0.1155058E-01	0.9364360E-03
0.5000114E-01	0.2474529E 00	0.7886249E-01	0.1168662E-01	0.9632134E-03
0.1000023E-01	0.2354717E 00	0.7903999E-01	0.1181771E-01	0.9893202E-03
0.0	0.2231483E 00	0.7867420E-01	0.1194140E-01	0.1014262E-02
0.0	0.2104930E 00	0.7810438E-01	0.1206053E-01	0.1038530E-02
0.0	0.1975491E 00	0.7734889E-01	0.1217287E-01	0.1061665E-02
0.0	0.1843243E 00	0.7640886E-01	0.1228101E-01	0.1084134E-02
0.0	0.1708542E 00	0.7530075E-01	0.1238292E-01	0.1105516E-02
0.0	0.1571445E 00	0.7402593E-01	0.1248096E-01	0.1126248E-02
0.0	0.1432250E 00	0.7259965E-01	0.1257330E-01	0.1145945E-02
0.0	0.1290998E 00	0.7102311E-01	0.1266214E-01	0.1165019E-02
0.0	0.1147941E 00	0.6931043E-01	0.1274575E-01	0.1183118E-02
0.0	0.1016320E 00	0.6746250E-01	0.1282105E-01	0.1200620E-02
0.0	0.9411842E-01	0.6549227E-01	0.1286312E-01	0.1217203E-02
0.0	0.8721364E-01	0.6340033E-01	0.1288365E-01	0.1233228E-02
0.0	0.8103186E-01	0.6119857E-01	0.1288179E-01	0.1248393E-02
0.0	0.7531589E-01	0.5888793E-01	0.1285702E-01	0.1263033E-02
0.0	0.7016420E-01	0.5647861E-01	0.1280914E-01	0.1276876E-02
0.0	0.6537706E-01	0.5397129E-01	0.1273783E-01	0.1290228E-02
0.0	0.6103885E-01	0.5137560E-01	0.1264345E-01	0.1302843E-02
0.0	0.5699090E-01	0.4869222E-01	0.1252573E-01	0.1315002E-02
0.0	0.5330572E-01	0.4592937E-01	0.1238539E-01	0.1326479E-02
0.0	0.4985565E-01	0.4319631E-01	0.1222230E-01	0.1337375E-02
0.0	0.4670316E-01	0.4093200E-01	0.1203736E-01	0.1346559E-02
0.0	0.4374346E-01	0.3876021E-01	0.1183052E-01	0.1354278E-02
0.0	0.4103064E-01	0.3672235E-01	0.1160285E-01	0.1360251E-02
0.0	0.3847763E-01	0.3476894E-01	0.1135435E-01	0.1364438E-02
0.0	0.3613150E-01	0.3293722E-01	0.1108619E-01	0.1366650E-02
0.0	0.3391917E-01	0.3118228E-01	0.1079832E-01	0.1366854E-02
0.0	0.3188170E-01	0.2953749E-01	0.1049203E-01	0.1364928E-02
0.0	0.2995713E-01	0.2796216E-01	0.1016730E-01	0.1360847E-02
0.0	0.2818130E-01	0.2648618E-01	0.9825367E-02	0.1354550E-02
0.0	0.2650142E-01	0.2507285E-01	0.9471346E-02	0.1346012E-02
0.0	0.2494891E-01	0.2374899E-01	0.9131275E-02	0.1335230E-02
0.0	0.2347636E-01	0.2248155E-01	0.8790996E-02	0.1322179E-02
0.0	0.2211738E-01	0.2129447E-01	0.8457012E-02	0.1306893E-02
0.0	0.2082679E-01	0.2015810E-01	0.8125447E-02	0.1289356E-02
0.0	0.1963087E-01	0.1909391E-01	0.7802580E-02	0.1269632E-02
0.0	0.1849572E-01	0.1807523E-01	0.7483952E-02	0.1247705E-02

0.0	0.1744265E-01	0.1712132E-01	0.7175572E-02	0.1225662E-02
0.0	0.1644224E-01	0.1620824E-01	0.6672620E-02	0.1197492E-02
0.0	0.1551327E-01	0.1535322E-01	0.6580800E-02	0.1169235E-02
0.0	0.1463005E-01	0.1453480E-01	0.6295178E-02	0.1139225E-02
0.0	0.1380920E-01	0.1376845E-01	0.6021108E-02	0.1108392E-02
0.0	0.1302823E-01	0.1303489E-01	0.5755618E-02	0.1076544E-02
0.0	0.1230184E-01	0.1234799E-01	0.5497746E-02	0.1044246E-02
0.0	0.1161032E-01	0.1169048E-01	0.5248636E-02	0.1011342E-02
0.0	0.1096670E-01	0.1107478E-01	0.5010944E-02	0.9784177E-03
0.0	0.1035361E-01	0.1048539E-01	0.4779994E-02	0.9452181E-03
0.0	0.9782631E-02	0.9933472E-02	0.4560120E-02	0.9123483E-03
0.0	0.9238459E-02	0.9405129E-02	0.4346821E-02	0.8794668E-03
0.0	0.8731391E-02	0.8910437E-02	0.4144125E-02	0.8471792E-03
0.0	0.8247897E-02	0.8436680E-02	0.3947772E-02	0.8150865E-03
0.0	0.7797144E-02	0.7993087E-02	0.3761475E-02	0.7837820E-03
0.0	0.7367183E-02	0.7568415E-02	0.3581232E-02	0.7528265E-03
0.0	0.6966129E-02	0.7170681E-02	0.3410451E-02	0.7227943E-03
0.0	0.6583445E-02	0.6789900E-02	0.3245408E-02	0.6932237E-03
0.0	0.6226342E-02	0.6433263E-02	0.3089200E-02	0.6646030E-03
0.0	0.5885486E-02	0.6091811E-02	0.2938380E-02	0.6366407E-03
0.0	0.5567297E-02	0.5771991E-02	0.2795789E-02	0.6096787E-03
0.0	0.5263489E-02	0.5465776E-02	0.2658231E-02	0.5833025E-03
0.0	0.4979771E-02	0.5178958E-02	0.2528293E-02	0.5580056E-03
0.0	0.4708797E-02	0.4904319E-02	0.2403034E-02	0.5333205E-03
0.0	0.4455682E-02	0.4647072E-02	0.2284810E-02	0.5097103E-03
0.0	0.4213862E-02	0.4400756E-02	0.2170914E-02	0.4867208E-03
0.0	0.3987927E-02	0.4170023E-02	0.2063490E-02	0.4647840E-03
0.0	0.3772037E-02	0.3949076E-02	0.1960081E-02	0.4434632E-03
0.0	0.3570262E-02	0.3742113E-02	0.1862570E-02	0.4231599E-03
0.0	0.3377392E-02	0.3543918E-02	0.1768745E-02	0.4034587E-03
0.0	0.3197102E-02	0.3358240E-02	0.1680361E-02	0.3847298E-03
0.0	0.3024759E-02	0.3180437E-02	0.1595342E-02	0.3665832E-03
0.0	0.2863597E-02	0.3013850E-02	0.1515289E-02	0.3493580E-03
0.0	0.2709518E-02	0.2854327E-02	0.1438316E-02	0.3326889E-03
0.0	0.2565415E-02	0.2704866E-02	0.1365874E-02	0.3168879E-03
0.0	0.2427613E-02	0.2561734E-02	0.1296242E-02	0.3016125E-03
0.0	0.2298708E-02	0.2427628E-02	0.1230736E-02	0.2871503E-03
0.0	0.2175426E-02	0.2299198E-02	0.1167791E-02	0.2731821E-03
0.0	0.2060082E-02	0.2178871E-02	0.1108596E-02	0.2599724E-03
0.0	0.1949753E-02	0.2063635E-02	0.1051734E-02	0.2472235E-03
0.0	0.1846515E-02	0.1955658E-02	0.9982761E-03	0.2351779E-03
0.0	0.1747751E-02	0.1852246E-02	0.9469385E-03	0.2235624E-03
0.0	0.1655323E-02	0.1755350E-02	0.8986869E-03	0.2125964E-03
0.0	0.1566892E-02	0.1662550E-02	0.8523613E-03	0.2020286E-03
0.0	0.1484120E-02	0.1575591E-02	0.8088320E-03	0.1920588E-03
0.0	0.1404920E-02	0.1492308E-02	0.7670489E-03	0.1824564E-03
0.0	0.1330782E-02	0.1414266E-02	0.7277974E-03	0.1734033E-03
0.0	0.1259834E-02	0.1339522E-02	0.6901266E-03	0.1646897E-03
0.0	0.1193414E-02	0.1269481E-02	0.6547463E-03	0.1564783E-03
0.0	0.1129849E-02	0.1202398E-02	0.6207975E-03	0.1485785E-03
0.0	0.1070333E-02	0.1139536E-02	0.5889190E-03	0.1411381E-03
0.0	0.1013370E-02	0.1079327E-02	0.5583342E-03	0.1339827E-03
0.0	0.9600311E-03	0.1022906E-02	0.5296194E-03	0.1272470E-03
0.0	0.9089762E-03	0.9688663E-03	0.5020755E-03	0.1207717E-03
0.0	0.8611663E-03	0.9182252E-03	0.4762188E-03	0.1146787E-03
0.0	0.8154011E-03	0.8697198E-03	0.4514190E-03	0.1088231E-03
0.0	0.7725405E-03	0.8242659E-03	0.4281427E-03	0.1033155E-03
0.0	0.7315096E-03	0.7807293E-03	0.4058199E-03	0.9802400E-04
0.0	0.6930823E-03	0.7399293E-03	0.3848709E-03	0.9304851E-04
0.0	0.6562942E-03	0.7008505E-03	0.3647823E-03	0.8826978E-04
0.0	0.6218357E-03	0.6642274E-03	0.3459335E-03	0.8377782E-04
0.0	0.5883459E-03	0.6291501E-03	0.3278593E-03	0.7946431E-04
0.0	0.5579439E-03	0.5962767E-03	0.3109016E-03	0.7541088E-04
0.0	0.5283575E-03	0.5647894E-03	0.2946442E-03	0.7151967E-04
0.0	0.5006436E-03	0.5352814E-03	0.2793919E-03	0.6786361E-04
0.0	0.4741063E-03	0.5070167E-03	0.2647706E-03	0.6435455E-04
0.0	0.4492474E-03	0.4805278E-03	0.2510541E-03	0.6105857E-04
0.0	0.4254445E-03	0.4551562E-03	0.2379060E-03	0.5789560E-04
0.0	0.4031456E-03	0.4313777E-03	0.2255731E-03	0.5492511E-04
0.0	0.3817929E-03	0.4086019E-03	0.2137514E-03	0.5207511E-04
0.0	0.3617888E-03	0.3872574E-03	0.2026638E-03	0.4939907E-04
0.0	0.3426326E-03	0.3668121E-03	0.1920364E-03	0.4683190E-04
0.0	0.3246865E-03	0.3476513E-03	0.1820694E-03	0.4442182E-04
0.0	0.3075004E-03	0.3292975E-03	0.1725165E-03	0.4211014E-04
0.0	0.2913983E-03	0.3120967E-03	0.1635580E-03	0.3994025E-04
0.0	0.2759784E-03	0.2956206E-03	0.1549721E-03	0.3785919E-04
0.0	0.2615310E-03	0.2801802E-03	0.1469209E-03	0.3590601E-04
0.0	0.2476955E-03	0.2653906E-03	0.1392049E-03	0.3403302E-04
0.0	0.2347327E-03	0.2515302E-03	0.1319697E-03	0.3227538E-04
0.0	0.2223174E-03	0.2382537E-03	0.1250361E-03	0.3058999E-04
0.0	0.2106850E-03	0.2258116E-03	0.1185348E-03	0.2900868E-04
0.0	0.1995444E-03	0.2138928E-03	0.1123046E-03	0.2749241E-04
0.0	0.1891052E-03	0.2027230E-03	0.1064630E-03	0.2606995E-04
0.0	0.1791080E-03	0.1920232E-03	0.1008655E-03	0.2470617E-04
	0.35504114E-02	0.23994931E 03		

0.9999996E-01	0.0	0.0	0.0	0.0
0.1999999E 00	0.0	0.0	0.0	0.0
0.3000000E 00	0.0	0.0	0.0	0.0
0.4000000E 00	0.0	0.0	0.0	0.0
0.5000000E 00	0.2844407E-02	0.0	0.0	0.0
0.5999999E 00	0.1879156E-01	0.0	0.0	0.0
0.6999999E 00	0.3792577E-01	0.0	0.0	0.0
0.8000000E 00	0.5939355E-01	0.0	0.0	0.0
0.9000000E 00	0.8290714E-01	0.0	0.0	0.0
0.9999999E 00	0.1079699E 00	0.2455362E-03	0.0	0.0
0.1000000E 01	0.1344283E 00	0.1845161E-02	0.0	0.0
0.1000000E 01	0.1619688E 00	0.4198845E-02	0.0	0.0
0.1000000E 01	0.1905027E 00	0.7180419E-02	0.0	0.0
0.1000000E 01	0.2198191E 00	0.1077433E-01	0.0	0.0
0.1000000E 01	0.2470189E 00	0.1487615E-01	0.1239311E-04	0.0
0.1000000E 01	0.2614521E 00	0.1947188E-01	0.1102946E-03	0.0
0.1000000E 01	0.2737297E 00	0.2447653E-01	0.2908900E-03	0.0
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0.1000000E 01	0.2923729E 00	0.3560872E-01	0.9011882E-03	0.0
0.1000000E 01	0.2995567E 00	0.4141234E-01	0.1331212E-02	0.4376253E-06
0.1000000E 01	0.3058762E 00	0.4606754E-01	0.1846207E-02	0.4734525E-05
0.1000000E 01	0.3111886E 00	0.5034061E-01	0.2436449E-02	0.1460642E-04
0.1000000E 01	0.3159062E 00	0.5409230E-01	0.3104056E-02	0.3075924E-04
0.1000000E 01	0.3199098E 00	0.5753117E-01	0.3838917E-02	0.5457773E-04
0.1000000E 01	0.3234891E 00	0.6054679E-01	0.4629519E-02	0.8598773E-04
0.1000000E 01	0.3265480E 00	0.6330931E-01	0.5386773E-02	0.1259152E-03
0.1000000E 01	0.3292966E 00	0.6573075E-01	0.6132867E-02	0.1737877E-03
0.1000000E 01	0.3316580E 00	0.6794840E-01	0.6835446E-02	0.2301879E-03
0.1000000E 01	0.3337882E 00	0.6989199E-01	0.7513233E-02	0.2942751E-03
0.1000000E 01	0.3356258E 00	0.7167202E-01	0.8139219E-02	0.3659567E-03
0.9000015E 00	0.3372883E 00	0.7323217E-01	0.8735254E-02	0.4404299E-03
0.8000011E 00	0.3387271E 00	0.7466120E-01	0.9278897E-02	0.5174584E-03
0.7000008E 00	0.3400321E 00	0.7591391E-01	0.9791996E-02	0.5934977E-03
0.6000004E 00	0.3411646E 00	0.7706165E-01	0.1025600E-01	0.6694547E-03
0.5000000E 00	0.3393497E 00	0.7806784E-01	0.1069126E-01	0.7420725E-03
0.4000015E 00	0.3242973E 00	0.7898980E-01	0.1108243E-01	0.8130127E-03
0.3000011E 00	0.3059774E 00	0.7979846E-01	0.1144769E-01	0.8794048E-03
0.2000008E 00	0.2852185E 00	0.8053941E-01	0.1177442E-01	0.9432849E-03
0.1000004E 00	0.2623518E 00	0.8118945E-01	0.1207845E-01	0.1002188E-02
0.2000046E-01	0.2378528E 00	0.8153981E-01	0.1234943E-01	0.1058253E-02
0.0	0.2119087E 00	0.8046299E-01	0.1260090E-01	0.1109396E-02
0.0	0.1848178E 00	0.7858855E-01	0.1282441E-01	0.1157690E-02
0.0	0.1566944E 00	0.7602751E-01	0.1303138E-01	0.1201394E-02
0.0	0.1277373E 00	0.7281917E-01	0.1321494E-01	0.1242419E-02
0.0	0.1008652E 00	0.6905580E-01	0.1337225E-01	0.1279318E-02
0.0	0.8671904E-01	0.6477028E-01	0.1342458E-01	0.1313796E-02
0.0	0.7470369E-01	0.6003819E-01	0.1338267E-01	0.1344665E-02
0.0	0.6495261E-01	0.5488608E-01	0.1324429E-01	0.1373406E-02
0.0	0.5650090E-01	0.4937522E-01	0.1300806E-01	0.1399041E-02
0.0	0.4950157E-01	0.4377278E-01	0.1267787E-01	0.1422410E-02
0.0	0.4335088E-01	0.3929427E-01	0.1225486E-01	0.1439284E-02
0.0	0.3818653E-01	0.3518330E-01	0.1174579E-01	0.1449028E-02
0.0	0.3360450E-01	0.3157399E-01	0.1115289E-01	0.1450277E-02
0.0	0.2971980E-01	0.2826571E-01	0.1048391E-01	0.1442557E-02
0.0	0.2624922E-01	0.2536467E-01	0.9753916E-02	0.1425403E-02

0.0	0.2328580E-01	0.2270732E-01	0.9050071E-02	0.1398642E-02
0.0	0.2062456E-01	0.2037829E-01	0.8353092E-02	0.1362413E-02
0.0	0.1833995E-01	0.1824535E-01	0.7693768E-02	0.1316753E-02
0.0	0.1628009E-01	0.1637619E-01	0.7055741E-02	0.1262154E-02
0.0	0.1450432E-01	0.1466441E-01	0.6464761E-02	0.1199218E-02
0.0	0.1289820E-01	0.1316422E-01	0.5900890E-02	0.1132462E-02
0.0	0.1150899E-01	0.1179018E-01	0.5385555E-02	0.1062539E-02
0.0	0.1024932E-01	0.1058577E-01	0.4898451E-02	0.9927663E-03
0.0	0.9156872E-02	0.9482451E-02	0.4457332E-02	0.9225754E-03
0.0	0.8164279E-02	0.8515134E-02	0.4043084E-02	0.8550400E-03
0.0	0.7301550E-02	0.7628836E-02	0.3670394E-02	0.7887736E-03
0.0	0.6516390E-02	0.6851595E-02	0.3322113E-02	0.7264987E-03
0.0	0.5832747E-02	0.6139319E-02	0.3010309E-02	0.6664030E-03
0.0	0.5209699E-02	0.5514544E-02	0.2719996E-02	0.6109324E-03
0.0	0.4666425E-02	0.4941877E-02	0.2461065E-02	0.5573283E-03
0.0	0.4170742E-02	0.4439469E-02	0.2220661E-02	0.5093811E-03
0.0	0.3738006E-02	0.3978882E-02	0.2006878E-02	0.4635626E-03
0.0	0.3342797E-02	0.3574735E-02	0.1808830E-02	0.4220363E-03
0.0	0.2997421E-02	0.3204162E-02	0.1633118E-02	0.3830132E-03
0.0	0.2681763E-02	0.2878930E-02	0.1470627E-02	0.3478734E-03
0.0	0.2405664E-02	0.2580685E-02	0.1326728E-02	0.3150094E-03
0.0	0.2153151E-02	0.2318895E-02	0.1193844E-02	0.2855628E-03
0.0	0.1932132E-02	0.2078810E-02	0.1076338E-02	0.2581265E-03
0.0	0.1729679E-02	0.1868041E-02	0.9679503E-03	0.2336375E-03
0.0	0.1552746E-02	0.1674723E-02	0.8722197E-03	0.2108865E-03
0.0	0.1390580E-02	0.1504996E-02	0.7839997E-03	0.1906421E-03
0.0	0.1248484E-02	0.1349308E-02	0.7061569E-03	0.1718776E-03
0.0	0.1118343E-02	0.1212608E-02	0.6344751E-03	0.1552202E-03
0.0	0.1004263E-02	0.1087207E-02	0.5712761E-03	0.1398094E-03
0.0	0.8997458E-03	0.9770931E-03	0.5131145E-03	0.1261554E-03
0.0	0.8080960E-03	0.8760744E-03	0.4618680E-03	0.1135422E-03
0.0	0.7241073E-03	0.7873653E-03	0.4147307E-03	0.1023843E-03
0.0	0.6504352E-03	0.7059802E-03	0.3732201E-03	0.9208918E-04
0.0	0.5829085E-03	0.6345084E-03	0.3350533E-03	0.8299350E-04
0.0	0.5236617E-03	0.5689352E-03	0.3014572E-03	0.7460937E-04
0.0	0.4693451E-03	0.5113466E-03	0.2705788E-03	0.6720942E-04
0.0	0.4216810E-03	0.4585097E-03	0.2434076E-03	0.6039385E-04
0.0	0.3779761E-03	0.4121044E-03	0.2184411E-03	0.5438342E-04
0.0	0.3396173E-03	0.3695274E-03	0.1964790E-03	0.4885125E-04
0.0	0.3044405E-03	0.3321324E-03	0.1763031E-03	0.4397596E-04
0.0	0.2735618E-03	0.2978207E-03	0.1585595E-03	0.3949087E-04
0.0	0.2452426E-03	0.2676852E-03	0.1422625E-03	0.3554061E-04
0.0	0.2203809E-03	0.2400350E-03	0.1279330E-03	0.3190819E-04
0.0	0.1975767E-03	0.2157496E-03	0.1147736E-03	0.2871039E-04
0.0	0.1776551E-03	0.1934659E-03	0.1032049E-03	0.2577090E-04
0.6666666E-01	0.0	0.0	0.0	0.0
0.1333333E 00	0.0	0.0	0.0	0.0
0.1999999E 00	0.0	0.0	0.0	0.0
0.2666667E 00	0.0	0.0	0.0	0.0
0.3333333E 00	0.0	0.0	0.0	0.0
0.4000000E 00	0.0	0.0	0.0	0.0
0.4666666E 00	0.0	0.0	0.0	0.0
0.5333333E 00	0.7717032E-02	0.0	0.0	0.0
0.5999999E 00	0.1681217E-01	0.0	0.0	0.0
0.6666666E 00	0.3120891E-01	0.0	0.0	0.0
0.7333332E 00	0.4481221E-01	0.0	0.0	0.0
0.8000000E 00	0.5937929E-01	0.0	0.0	0.0
0.8666666E 00	0.7485205E-01	0.0	0.0	0.0
0.9333333E 00	[illegible]	0.0	0.0	0.0
0.9866666E 00	0.1079620E 00	0.1506981E-03	0.0	0.0

0.1000000E 01	0.1254376E 00	0.1068126E-02	0.0	0.0
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0.1000000E 01	0.1619442E 00	0.3877357E-02	0.0	0.0
0.1000000E 01	0.1808654E 00	0.5726505E-02	0.0	0.0
0.1000000E 01	0.2001521E 00	0.7832032E-02	0.0	0.0
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0.1000000E 01	0.2616420E 00	0.1860119E-01	0.1105789E-03	0.0
0.1000000E 01	0.2698745E 00	0.2180109E-01	0.2197648E-03	0.0
0.1000000E 01	0.2772878E 00	0.2516817E-01	0.3672752E-03	0.0
0.1000000E 01	0.2837778E 00	0.2870364E-01	0.5508845E-03	0.0
0.1000000E 01	0.2896705E 00	0.3238367E-01	0.7738250E-03	0.0
0.1000000E 01	0.2948750E 00	0.3620933E-01	0.1032900E-02	0.0
0.1000000E 01	0.2996296E 00	0.4000920E-01	0.1330581E-02	0.2381923E-06
0.1000000E 01	0.3038581E 00	0.4314208E-01	0.1663168E-02	0.2259062E-05
0.1000000E 01	0.3077401E 00	0.4609498E-01	0.2032574E-02	0.6390563E-05
0.1000000E 01	0.3112110E 00	0.4881360E-01	0.2434905E-02	0.1285280E-04
0.1000000E 01	0.3144097E 00	0.5137375E-01	0.2871647E-02	0.2217901E-04
0.1000000E 01	0.3172818E 00	0.5372848E-01	0.3338929E-02	0.3440006E-04
0.1000000E 01	0.3199369E 00	0.5594487E-01	0.3837905E-02	0.4996452E-04
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0.1000000E 01	0.3245464E 00	0.5989913E-01	0.4882582E-02	0.9114406E-04
0.1000000E 01	0.3265496E 00	0.6166065E-01	0.5391341E-02	0.1169072E-03
0.1000000E 01	0.3284101E 00	0.6331760E-01	0.5886540E-02	0.1463465E-03
0.1000000E 01	0.3300953E 00	0.6484014E-01	0.6372184E-02	0.1791698E-03
0.1000000E 01	0.3316631E 00	0.6627214E-01	0.6839201E-02	0.2156202E-03
0.1000000E 01	0.3330854E 00	0.6758773E-01	0.7293418E-02	0.2553640E-03
0.1000000E 01	0.3344112E 00	0.6882507E-01	0.7726446E-02	0.2985983E-03
0.1000000E 01	0.3356159E 00	0.6996185E-01	0.8145049E-02	0.3447311E-03
0.9333344E 00	0.3367406E 00	0.7103097E-01	0.8541562E-02	0.3922130E-03
0.8666668E 00	0.3377639E 00	0.7201326E-01	0.8923132E-02	0.4408460E-03
0.8000011E 00	0.3387197E 00	0.7293707E-01	0.9282771E-02	0.4898286E-03
0.7333336E 00	0.3395910E 00	0.7378602E-01	0.9627618E-02	0.5391040E-03
0.6666679E 00	0.3404055E 00	0.7458454E-01	0.9951424E-02	0.5878231E-03
0.6000004E 00	0.3411487E 00	0.7531822E-01	0.1026103E-01	0.6361972E-03
0.5333347E 00	0.3418442E 00	0.7600820E-01	0.1055084E-01	0.6833742E-03
0.4666672E 00	0.3347623E 00	0.7664257E-01	0.1082734E-01	0.7297618E-03
0.4000015E 00	0.3242620E 00	0.7723904E-01	0.1108548E-01	0.7745333E-03
0.3333340E 00	0.3124082E 00	0.7778734E-01	0.1133128E-01	0.8182246E-03
0.2666683E 00	0.2993134E 00	0.7830304E-01	0.1156031E-01	0.8600522E-03
0.2000008E 00	0.2852126E 00	0.7877719E-01	0.1177805E-01	0.9006299E-03
0.1333351E 00	0.2701764E 00	0.7922322E-01	0.1198057E-01	0.9392269E-03
0.6666756E-01	0.2543705E 00	0.7963324E-01	0.1217284E-01	0.9764936E-03
0.1333427E-01	0.2378411E 00	0.7986832E-01	0.1235142E-01	0.1011756E-02
0.0	0.2207085E 00	0.7930541E-01	0.1252078E-01	0.1045673E-02
0.0	0.2030045E 00	0.7838142E-01	0.1267787E-01	0.1077629E-02
0.0	0.1848192E 00	0.7713646E-01	0.1282670E-01	0.1108269E-02
0.0	0.1661755E 00	0.7557613E-01	0.1296461E-01	0.1137038E-02
0.0	0.1471425E 00	0.7373577E-01	0.1309516E-01	0.1164550E-02
0.0	0.1277372E 00	0.7162106E-01	0.1321601E-01	0.1190307E-02
0.0	0.1080135E 00	0.6926310E-01	0.1333034E-01	0.1214883E-02
0.0	0.9582859E-01	0.6666720E-01	0.1340064E-01	0.1237837E-02
0.0	0.8650440E-01	0.6386054E-01	0.1342551E-01	0.1259696E-02
0.0	0.7844925E-01	0.6084782E-01	0.1340872E-01	0.1280072E-02
0.0	0.7119846E-01	0.5765261E-01	0.1334857E-01	0.1299446E-02
0.0	0.6486118E-01	0.5427910E-01	0.1324561E-01	0.1317472E-02
0.0	0.5910924E-01	0.5074779E-01	0.1309888E-01	0.1334590E-02
0.0	0.5403700E-01	0.4706239E-01	0.1291015E-01	0.1350493E-02
0.0	0.4940331E-01	0.4339129E-01	0.1267891E-01	0.1365342E-02
0.0	0.4528880E-01	0.4037984E-01	0.1240760E-01	0.1377318E-02

0.0	0.4151155E-01	0.3753845E-01	0.1209607E-01	0.1386453E-02
0.0	0.3813919E-01	0.3492499E-01	0.1174710E-01	0.1392284E-02
0.0	0.3503083E-01	0.3246142E-01	0.1136073E-01	0.1394600E-02
0.0	0.3224377E-01	0.3019755E-01	0.1093988E-01	0.1393156E-02
0.0	0.2966670E-01	0.2806486E-01	0.1048473E-01	0.1387791E-02
0.0	0.2734792E-01	0.2610619E-01	0.9998150E-02	0.1378434E-02
0.0	0.2519827E-01	0.2426171E-01	0.9518549E-02	0.1364970E-02
0.0	0.2325851E-01	0.2256833E-01	0.9044886E-02	0.1347454E-02
0.0	0.2145632E-01	0.2097398E-01	0.8582812E-02	0.1325812E-02
0.0	0.1982616E-01	0.1951051E-01	0.8127660E-02	0.1300189E-02
0.0	0.1830886E-01	0.1813279E-01	0.7689420E-02	0.1270547E-02
0.0	0.1693365E-01	0.1686828E-01	0.7261693E-02	0.1237083E-02
0.0	0.1565164E-01	0.1567790E-01	0.6853659E-02	0.1199786E-02
0.0	0.1448767E-01	0.1458537E-01	0.6458055E-02	0.1159127E-02
0.0	0.1340113E-01	0.1355687E-01	0.6083228E-02	0.1116819E-02
0.0	0.1241320E-01	0.1261289E-01	0.5721636E-02	0.1072955E-02
0.0	0.1148992E-01	0.1172422E-01	0.5380817E-02	0.1028475E-02
0.0	0.1064936E-01	0.1090851E-01	0.5053282E-02	0.9833481E-03
0.0	0.9863019E-02	0.1014055E-01	0.4745826E-02	0.9385485E-03
0.0	0.9146344E-02	0.9435602E-02	0.4451249E-02	0.8937645E-03
0.0	0.8475304E-02	0.8771881E-02	0.4175607E-02	0.8500144E-03
0.0	0.7863127E-02	0.8162569E-02	0.3912158E-02	0.8067645E-03
0.0	0.7289499E-02	0.7588841E-02	0.3666307E-02	0.7649553E-03
0.0	0.6765727E-02	0.7062104E-02	0.3431806E-02	0.7239911E-03
0.0	0.6274629E-02	0.6566096E-02	0.3213436E-02	0.6847365E-03
0.0	0.5825900E-02	0.6110664E-02	0.3005507E-02	0.6465276E-03
0.0	0.5404901E-02	0.5681764E-02	0.2812234E-02	0.6101660E-03
0.0	0.5019952E-02	0.5287938E-02	0.2628457E-02	0.5749573E-03
0.0	0.4658628E-02	0.4917014E-02	0.2457898E-02	0.5416342E-03
0.0	0.4328053E-02	0.4576396E-02	0.2295917E-02	0.5095040E-03
0.0	0.4017621E-02	0.4255589E-02	0.2145779E-02	0.4792304E-03
0.0	0.3733472E-02	0.3960956E-02	0.2003336E-02	0.4501399E-03
0.0	0.3466531E-02	0.3683442E-02	0.1871454E-02	0.4228314E-03
0.0	0.3222057E-02	0.3428555E-02	0.1746439E-02	0.3966643E-03
0.0	0.2992318E-02	0.3188447E-02	0.1630804E-02	0.3721751E-03
0.0	0.2781844E-02	0.2967906E-02	0.1521272E-02	0.3487645E-03
0.0	0.2583978E-02	0.2760144E-02	0.1420040E-02	0.3269115E-03
0.0	0.2402642E-02	0.2569297E-02	0.1324214E-02	0.3060619E-03
0.0	0.2232122E-02	0.2389507E-02	0.1235712E-02	0.2866404E-03
0.0	0.2075799E-02	0.2224347E-02	0.1151982E-02	0.2681424E-03
0.0	0.1928761E-02	0.2068753E-02	0.1074699E-02	0.2509428E-03
0.0	0.1793926E-02	0.1925810E-02	0.1001618E-02	0.2345836E-03
0.0	0.1667073E-02	0.1791134E-02	0.9342001E-03	0.2193965E-03
0.0	0.1550719E-02	0.1667407E-02	0.8704741E-03	0.2049698E-03
0.0	0.1441229E-02	0.1550833E-02	0.8117144E-03	0.1915938E-03
0.0	0.1340782E-02	0.1443730E-02	0.7561923E-03	0.1788999E-03
0.0	0.1246242E-02	0.1342816E-02	0.7050170E-03	0.1671445E-03
0.0	0.1159494E-02	0.1250098E-02	0.6566781E-03	0.1559987E-03
0.0	0.1077837E-02	0.1162736E-02	0.6121392E-03	0.1456865E-03
0.0	0.1002894E-02	0.1082468E-02	0.5700798E-03	0.1359170E-03
0.0	0.9323398E-03	0.1006834E-02	0.5313396E-03	0.1268854E-03
0.0	0.8675775E-03	0.9373396E-03	0.4947644E-03	0.1183348E-03
0.0	0.8065999E-03	0.8718553E-03	0.4610850E-03	0.1104360E-03
0.0	0.7506208E-03	0.8116863E-03	0.4292941E-03	0.1029623E-03
0.0	0.6979075E-03	0.7549883E-03	0.4000275E-03	0.9606249E-04
0.0	0.6495090E-03	0.7028915E-03	0.3724073E-03	0.8953712E-04
0.0	0.6039294E-03	0.6537992E-03	0.3469847E-03	0.8351638E-04
0.0	0.5620776E-03	0.6086892E-03	0.3229971E-03	0.7782481E-04
0.0	0.5226592E-03	0.5661803E-03	0.3009229E-03	0.7257600E-04

0.0	0.4864600E-03	0.5271202E-03	0.2800962E-03	0.6761594E-04
0.0	0.4523646E-03	0.4903108E-03	0.2609345E-03	0.6304364E-04
0.0	0.4210512E-03	0.4564873E-03	0.2428585E-03	0.5872420E-04
0.0	0.3915550E-03	0.4246135E-03	0.2262293E-03	0.5474407E-04
0.0	0.3644640E-03	0.3953243E-03	0.2105442E-03	0.5098509E-04
0.0	0.3389434E-03	0.3677222E-03	0.1961163E-03	0.4752245E-04
0.0	0.3155021E-03	0.3423586E-03	0.1825089E-03	0.4425309E-04
0.0	0.2934188E-03	0.3184567E-03	0.1699933E-03	0.4124231E-04
0.0	0.2731339E-03	0.2964921E-03	0.1581904E-03	0.3840021E-04
0.0	0.2540234E-03	0.2757944E-03	0.1473360E-03	0.3578358E-04
0.0	0.2364684E-03	0.2567740E-03	0.1371003E-03	0.3331396E-04
0.0	0.2199282E-03	0.2388504E-03	0.1276880E-03	0.3104079E-04
0.0	0.2047338E-03	0.2223796E-03	0.1188129E-03	0.2889571E-04
0.0	0.1904176E-03	0.2068577E-03	0.1106522E-03	0.2692164E-04
0.0	0.1772654E-03	0.1925932E-03	0.1029575E-03	0.2505910E-04
0.5000000E-01	0.0	0.0	0.0	0.0
0.9999996E-01	0.0	0.0	0.0	0.0
0.1500000E 00	0.0	0.0	0.0	0.0
0.1999999E 00	0.0	0.0	0.0	0.0
0.2500000E 00	0.0	0.0	0.0	0.0
0.3000000E 00	0.0	0.0	0.0	0.0
0.3499999E 00	0.0	0.0	0.0	0.0
0.4000000E 00	0.0	0.0	0.0	0.0
0.4499999E 00	0.0	0.0	0.0	0.0
0.5000000E 00	0.1321034E-02	0.0	0.0	0.0
0.5500000E 00	0.8377887E-02	0.0	0.0	0.0
0.5999999E 00	0.1633499E-01	0.0	0.0	0.0
0.6500000E 00	0.2502728E-01	0.0	0.0	0.0
0.6999999E 00	0.3442798E-01	0.0	0.0	0.0
0.7500000E 00	0.4441303E-01	0.0	0.0	0.0
0.8000000E 00	0.5496659E-01	0.0	0.0	0.0
0.8499999E 00	0.6599307E-01	0.0	0.0	0.0
0.9000000E 00	0.7748353E-01	0.0	0.0	0.0
0.9499999E 00	0.8936238E-01	0.0	0.0	0.0
0.9899999E 00	0.1016248E 00	0.1083283E-03	0.0	0.0
0.1000000E 01	0.1142105E 00	0.7417507E-03	0.0	0.0
0.1000000E 01	0.1271170E 00	0.1567815E-02	0.0	0.0
0.1000000E 01	0.1402954E 00	0.2563274E-02	0.0	0.0
0.1000000E 01	0.1537443E 00	0.3733363E-02	0.0	0.0
0.1000000E 01	0.1674239E 00	0.5056720E-02	0.0	0.0
0.1000000E 01	0.1813340E C0	0.6537829E-02	0.0	0.0
0.1000000E 01	0.1954411E 00	0.8157190E-02	0.0	0.0
0.1000000E 01	0.2097450E 00	0.9918757E-02	0.0	0.0
0.1000000E 01	0.2242184E 00	0.1180477E-01	0.0	0.0
0.1000000E 01	0.2375407E 00	0.1381867E-01	0.5070567E-05	0.0
0.1000000E 01	0.2452039E 00	0.1594450E-01	0.3864350E-04	0.0
0.1000000E 01	0.2522522E 00	0.1818518E-01	0.9081286E-04	0.0
0.1000000E 01	0.2585679E 00	0.2052644E-01	0.1610400E-03	0.0
0.1000000E 01	0.2644120E 00	0.2297085E-01	0.2515251E-03	0.0
0.1000000E 01	0.2696837E 00	0.2550543E-01	0.3611504E-03	0.0
0.1000000E 01	0.2745857E 00	0.2813262E-01	0.4917916E-03	0.0
0.1000000E 01	0.2790315E 00	0.3084071E-01	0.6419457E-03	0.0
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0.1000000E 01	0.2869637E 00	0.3649534E-01	0.1003881E-02	0.0
0.1000000E 01	0.2905061E 00	0.3932513E-01	0.1215307E-02	0.1607165E-06
0.1000000E 01	0.2937447E 00	0.4167993E-01	0.1445627E-02	0.1401719E-05
0.1000000E 01	0.2967870E 00	0.4393852E-01	0.1696036E-02	0.3724990E-05
0.1000000E 01	0.2995774E 00	0.4605766E-01	0.1964609E-02	0.7217894E-05
0.1000000E 01	0.3022044E 00	0.4808897E-01	0.2252384E-02	0.1213886E-04
0.1000000E 01	0.3046197E 00	0.4999368E-01	0.2557433E-02	0.1850646E-04

0.1000000E 01	0.3068980E 00	0.5181837E-01	0.2880663E-02	0.2655829E-04
0.1000000E 01	0.3089976E 00	0.5352866E-01	0.3220182E-02	0.3625485E-04
0.1000000E 01	0.3109815E 00	0.5516665E-01	0.3576786E-02	0.4781148E-04
0.1000000E 01	0.3128128E 00	0.5670145E-01	0.3948644E-02	0.6114223E-04
0.1000000E 01	0.3145461E 00	0.5817116E-01	0.4331373E-02	0.7644035E-04
0.1000000E 01	0.3161484E 00	0.5954764E-01	0.4698392E-02	0.9358450E-04
0.1000000E 01	0.3176667E 00	0.6086564E-01	0.5064506E-02	0.1127468E-03
0.1000000E 01	0.3190724E 00	0.6210001E-01	0.5422745E-02	0.1337800E-03
0.1000000E 01	0.3204058E 00	0.6328183E-01	0.5777583E-02	0.1568373E-03
0.1000000E 01	0.3216416E 00	0.6438833E-01	0.6122265E-02	0.1817536E-03
0.1000000E 01	0.3228149E 00	0.6544775E-01	0.6461833E-02	0.2086637E-03
0.1000000E 01	0.3239043E 00	0.6643981E-01	0.6789856E-02	0.2373929E-03
0.1000000E 01	0.3249389E 00	0.6738943E-01	0.7111639E-02	0.2680614E-03
0.1000000E 01	0.3259000E 00	0.6827867E-01	0.7421110E-02	0.3004870E-03
0.1000000E 01	0.3268140E 00	0.6913006E-01	0.7723689E-02	0.3346147E-03
0.9500008E 00	0.3276637E 00	0.6992698E-01	0.8013632E-02	0.3692608E-03
0.9000015E 00	0.3284723E 00	0.7068992E-01	0.8296363E-02	0.4047656E-03
0.8500004E 00	0.3292246E 00	0.7140434E-01	0.8566506E-02	0.4405011E-03
0.8000011E 00	0.3299409E 00	0.7208836E-01	0.8829344E-02	0.4767065E-03
0.7500000E 00	0.3306077E 00	0.7272875E-01	0.9079855E-02	0.5127268E-03
0.7000008E 00	0.3312430E 00	0.7334179E-01	0.9323120E-02	0.5488985E-03
0.6500015E 00	0.3318351E 00	0.7391602E-01	0.9554554E-02	0.5845579E-03
0.6000004E 00	0.3323989E 00	0.7446557E-01	0.9778913E-02	0.6201160E-03
0.5500011E 00	0.3329249E 00	0.7498032E-01	0.9992000E-02	0.6549151E-03
0.5000000E 00	0.3321052E 00	0.7547313E-01	0.1019832E-01	0.6894181E-03
0.4500008E 00	0.3255159E 00	0.7593453E-01	0.1039394E-01	0.7229832E-03
0.4000015E 00	0.3180048E 00	0.7637632E-01	0.1058311E-01	0.7561084E-03
0.3500004E 00	0.3097282E 00	0.7679015E-01	0.1076230E-01	0.7881739E-03
0.3000011E 00	0.3007243E 00	0.7718629E-01	0.1093538E-01	0.8196994E-03
0.2500000E 00	0.2911097E 00	0.7755738E-01	0.1109913E-01	0.8500898E-03
0.2000008E 00	0.2809093E 00	0.7791263E-01	0.1125716E-01	0.8798703E-03
0.1500015E 00	0.2702129E 00	0.7824528E-01	0.1140654E-01	0.9084775E-03
0.1000004E 00	0.2590374E 00	0.7856405E-01	0.1155058E-01	0.9364360E-03
0.5000114E-01	0.2474529E 00	0.7886249E-01	0.1168662E-01	0.9632134E-03
0.1000023E-01	0.2354717E 00	0.7903999E-01	0.1181771E-01	0.9893202E-03
0.0	0.2231483E 00	0.7867420E-01	0.1194140E-01	0.1014262E-02
0.0	0.2104930E 00	0.7810438E-01	0.1206053E-01	0.1038530E-02
0.0	0.1975491E 00	0.7734889E-01	0.1217287E-01	0.1061665E-02
0.0	0.1843243E 00	0.7640886E-01	0.1228101E-01	0.1084134E-02
0.0	0.1708542E 00	0.7530075E-01	0.1238292E-01	0.1105516E-02
0.0	0.1571445E 00	0.7402593E-01	0.1248096E-01	0.1126248E-02
0.0	0.1432250E 00	0.7259965E-01	0.1257330E-01	0.1145945E-02
0.0	0.1290998E 00	0.7102311E-01	0.1266214E-01	0.1165019E-02
0.0	0.1147941E 00	0.6931043E-01	0.1274575E-01	0.1183118E-02
0.0	0.1016320E 00	0.6746250E-01	0.1282105E-01	0.1200620E-02
0.0	0.9411842E-01	0.6549227E-01	0.1286312E-01	0.1217203E-02
0.0	0.8721364E-01	0.6340033E-01	0.1288365E-01	0.1233228E-02
0.0	0.8103186E-01	0.6119857E-01	0.1288179E-01	0.1248393E-02
0.0	0.7531589E-01	0.5888793E-01	0.1285702E-01	0.1263033E-02
0.0	0.7016420E-01	0.5647861E-01	0.1280914E-01	0.1276876E-02
0.0	0.6537706E-01	0.5397129E-01	0.1273785E-01	0.1290228E-02
0.0	0.6103885E-01	0.5137560E-01	0.1264345E-01	0.1302843E-02
0.0	0.5699090E-01	0.4869222E-01	0.1252573E-01	0.1315002E-02
0.0	0.5330572E-01	0.4592937E-01	0.1238539E-01	0.1326479E-02
0.0	0.4985565E-01	0.4319631E-01	0.1222230E-01	0.1337375E-02
0.0	0.4670316E-01	0.4093200E-01	0.1203736E-01	0.1346559E-02
0.0	0.4374346E-01	0.3876021E-01	0.1183052E-01	0.1354278E-02
0.0	0.4103064E-01	0.3672235E-01	0.1160285E-01	0.1360251E-02
0.0	0.3847763E-01	0.3476894E-01	0.1135435E-01	0.1364438E-02
0.0	0.3613150E-01	0.3293722E-01	0.1108619E-01	0.1366650E-02

0.0	0.3391917E-01	0.3118228E-01	0.1079832E-01	0.1366854E-02
0.0	0.3188170E-01	0.2953749E-01	0.1049203E-01	0.1364928E-02
0.0	0.2995713E-01	0.2796216E-01	0.1016730E-01	0.1360847E-02
0.0	0.2818130E-01	0.2648618E-01	0.9825367E-02	0.1354550E-02
0.0	0.2650142E-01	0.2507285E-01	0.9471346E-02	0.1346012E-02
0.0	0.2494891E-01	0.2374899E-01	0.9131275E-02	0.1335230E-02
0.0	0.2347836E-01	0.2248155E-01	0.8790996E-02	0.1322179E-02
0.0	0.2211738E-01	0.2129447E-01	0.8457012E-02	0.1306693E-02
0.0	0.2082679E-01	0.2015810E-01	0.8125447E-02	0.1289356E-02
0.0	0.1963087E-01	0.1909391E-01	0.7802580E-02	0.1269632E-02
0.0	0.1849572E-01	0.1807523E-01	0.7483952E-02	0.1247705E-02
0.0	0.1744265E-01	0.1712132E-01	0.7175572E-02	0.1223662E-02
0.0	0.1644224E-01	0.1620824E-01	0.6872620E-02	0.1197492E-02
0.0	0.1551327E-01	0.1535322E-01	0.6580800E-02	0.1169296E-02
0.0	0.1463005E-01	0.1453480E-01	0.6295178E-02	0.1139225E-02
0.0	0.1380920E-01	0.1376845E-01	0.6021108E-02	0.1108392E-02
0.0	0.1302823E-01	0.1303489E-01	0.5753618E-02	0.1076548E-02
0.0	0.1230184E-01	0.1234799E-01	0.5497746E-02	0.1044248E-02
0.0	0.1161032E-01	0.1169048E-01	0.5248636E-02	0.1011342E-02
0.0	0.1096670E-01	0.1107478E-01	0.5010944E-02	0.9784177E-03
0.0	0.1035361E-01	0.1048539E-01	0.4779994E-02	0.9452181E-03
0.0	0.9782631E-02	0.9933472E-02	0.4560120E-02	0.9123483E-03
0.0	0.9238459E-02	0.9405129E-02	0.4346821E-02	0.8794668E-03
0.0	0.8731391E-02	0.8910347E-02	0.4144125E-02	0.8471792E-03
0.0	0.8247897E-02	0.8436680E-02	0.3947772E-02	0.8150865E-03
0.0	0.7797144E-02	0.7993087E-02	0.3761475E-02	0.7837820E-03
0.0	0.7367183E-02	0.7568415E-02	0.3581232E-02	0.7528265E-03
0.0	0.6966129E-02	0.7170681E-02	0.3410451E-02	0.7227943E-03
0.0	0.6583445E-02	0.6789900E-02	0.3245408E-02	0.6932237E-03
0.0	0.6226342E-02	0.6433263E-02	0.3089200E-02	0.6646630E-03
0.0	0.5885486E-02	0.6091811E-02	0.2938380E-02	0.6366407E-03
0.0	0.5567297E-02	0.5771991E-02	0.2795789E-02	0.6096787E-03
0.0	0.5263489E-02	0.5465776E-02	0.2658231E-02	0.5833025E-03
0.0	0.4979771E-02	0.5178958E-02	0.2528293E-02	0.5580056E-03
0.0	0.4708797E-02	0.4904319E-02	0.2403034E-02	0.5333205E-03
0.0	0.4455682E-02	0.4647072E-02	0.2284810E-02	0.5097103E-03
0.0	0.4213862E-02	0.4400756E-02	0.2170914E-02	0.4867208E-03
0.0	0.3987927E-02	0.4170023E-02	0.2063490E-02	0.4647840E-03
0.0	0.3772037E-02	0.3949076E-02	0.1960061E-02	0.4434632E-03
0.0	0.3570262E-02	0.3742113E-02	0.1862570E-02	0.4231599E-03
0.0	0.3377392E-02	0.3543918E-02	0.1768745E-02	0.4034587E-03
0.0	0.3197102E-02	0.3358240E-02	0.1680361E-02	0.3847298E-03
0.0	0.3024759E-02	0.3180437E-02	0.1595342E-02	0.3665832E-03
0.0	0.2863597E-02	0.3013850E-02	0.1515289E-02	0.3493580E-03
0.0	0.2709518E-02	0.2854327E-02	0.1438316E-02	0.3326889E-03
0.0	0.2565415E-02	0.2704866E-02	0.1365874E-02	0.3168879E-03
0.0	0.2427613E-02	0.2561734E-02	0.1296242E-02	0.3016125E-03
0.0	0.2298708E-02	0.2427628E-02	0.1230736E-02	0.2871503E-03
0.0	0.2175426E-02	0.2299198E-02	0.1167791E-02	0.2731821E-03
0.0	0.2060082E-02	0.2178871E-02	0.1108596E-02	0.2599724E-03
0.0	0.1949753E-02	0.2063635E-02	0.1051734E-02	0.2472235E-03
0.0	0.1846515E-02	0.1955658E-02	0.9982761E-03	0.2351779E-03
0.0	0.1747751E-02	0.1852246E-02	0.9469385E-03	0.2235624E-03
0.0	0.1655323E-02	0.1755350E-02	0.8986869E-03	0.2125964E-03
0.0	0.1566892E-02	0.1662550E-02	0.8523613E-03	0.2020286E-03
0.0	0.1484120E-02	0.1575591E-02	0.8088320E-03	0.1920588E-03
0.0	0.1404920E-02	0.1492308E-02	0.7670489E-03	0.1824564E-03
0.0	0.1330782E-02	0.1414266E-02	0.7277974E-03	0.1734033E-03
0.0	0.1259834E-02	0.1339522E-02	0.6901266E-03	0.1646897E-03
0.0	0.1193414E-02	0.1269481E-02	0.6547463E-03	0.1564783E-03
0.0	0.1129849E-02	0.1202398E-02	0.6207975E-03	0.1485785E-03
0.0	0.1070333E-02	0.1139536E-02	0.5889190E-03	0.1411381E-03
0.0	0.1013370E-02	0.1079327E-02	0.5583342E-03	0.1339827E-03
0.0	0.9600311E-03	0.1022906E-02	0.5296194E-03	0.1272470E-03
0.0	0.9089762E-03	0.9688663E-03	0.5020755E-03	0.1207717E-03
0.0	0.8611663E-03	0.9182252E-03	0.4762188E-03	0.1146787E-03
0.0	0.8154011E-03	0.8697198E-03	0.4514190E-03	0.1088231E-03
0.0	0.7725405E-03	0.8242659E-03	0.4281427E-03	0.1033155E-03
0.0	0.7315096E-03	0.7807293E-03	0.4058199E-03	0.9802400E-04
0.0	0.6930823E-03	0.7399293E-03	0.3848709E-03	0.9304851E-04
0.0	0.6562942E-03	0.7008505E-03	0.3647823E-03	0.8826978E-04
0.0	0.6218357E-03	0.6642274E-03	0.3459335E-03	0.8377782E-04
0.0	0.5888459E-03	0.6291501E-03	0.3278593E-03	0.7946431E-04
0.0	0.5579439E-03	0.5962767E-03	0.3109016E-03	0.7541088E-04
0.0	0.5283575E-03	0.5647894E-03	0.2946442E-03	0.7151967E-04
0.0	0.5006436E-03	0.5352814E-03	0.2793919E-03	0.6786361E-04
0.0	0.4741063E-03	0.5070167E-03	0.2647706E-03	0.6435455E-04
0.0	0.4492474E-03	0.4805278E-03	0.2510541E-03	0.6105857E-04
0.0	0.4254445E-03	0.4551562E-03	0.2379060E-03	0.5789560E-04
0.0	0.4031456E-03	0.4313777E-03	0.2255731E-03	0.5492511E-04
0.0	0.3817929E-03	0.4086019E-03	0.2137514E-03	0.5207511E-04
0.0	0.3617888E-03	0.3872574E-03	0.2026638E-03	0.4939907E-04
0.0	0.3426326E-03	0.3668121E-03	0.1920364E-03	0.4683190E-04
0.0	0.3246865E-03	0.3476513E-03	0.1820694E-03	0.4442182E-04
0.0	0.3075004E-03	0.3292975E-03	0.1725165E-03	0.4211014E-04
0.0	0.2913983E-03	0.3120967E-03	0.1635580E-03	0.3994025E-04
0.0	0.2759784E-03	0.2956206E-03	0.1549721E-03	0.3785919E-04
0.0	0.2615310E-03	0.2801802E-03	0.1469209E-03	0.3590601E-04
0.0	0.2476955E-03	0.2653906E-03	0.1392049E-03	0.3403302E-04
0.0	0.2347327E-03	0.2515302E-03	0.1319697E-03	0.3227538E-04
0.0	0.2223174E-03	0.2382537E-03	0.1250361E-03	0.3058999E-04
0.0	0.2106850E-03	0.2258116E-03	0.1185348E-03	0.2900868E-04
0.0	0.1995444E-03	0.2138928E-03	0.1123046E-03	0.2749241E-04
0.0	0.1891052E-03	0.2027230E-03	0.1064630E-03	0.2606995E-04
0.0	0.1791080E-03	0.1920232E-03	0.1008655E-03	0.2470617E-04
	0.14508043E-02	0.23995053E 03		

CHAPTER 6

CONCLUSION

A method for the evaluation of the transient response of a nonuniform lossy transmission line has been presented in this thesis. The pair of partial differential equations which describes the voltage and current relations on the line is transformed into a pair of ordinary differential equations on two characteristic curves using the method of characteristics.

A stepped line approximation is used to analyze the transient response of the given nonuniform line. The concept of electrical length is employed in dividing the line into a number of equal delay sections. The set of difference equations describing the set stepped line is suitable for digital computer solution.

The main advantage of this method is that numerical techniques such as the Runge-Kutta method are entirely avoided.

The computational procedure involving the use of a digital computer is illustrated for a specific distributions of L(x), C(x), r(x) and g(x).

There is still much work which has to be done in this area.

For instance, it will be worth while to investigate the possibility of obtaining an estimate of the proper number of sections to achieve a given accuracy in the case of nonuniform lines.

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APPENDIX A

BASIC EQUATIONS FOR A UNIFORM LOSSLESS LINE

Consider a uniform lossless transmission line of length 1 as shown in figure (A.1). The following two equations hold true for the voltage and current values w.r.t. the position on the line x and time.

$$\frac{\partial^2 V(x,s)}{\partial x^2} = s^2 \, L \, C \, V \, (x,s) \tag{A-1}$$

$$\frac{\partial^2 I(x,s)}{\partial x^2} = s^2 \, L \, C \, I \, (x,s) \tag{A-2}$$

Where L and C are the inductance and capacitance per unit length of the line respectively, and s being the Laplace operator.

The solution of A-1 and A-2 takes the form

$$V(x,s) = F(s) \, e^{\frac{-sx}{v}} + G(s) \, e^{\frac{sx}{v}} \tag{A-3}$$

$$\rho \, I(x,s) = F(s) \, e^{\frac{-sx}{v}} - G(s) \, e^{\frac{sx}{v}} \tag{A-4}$$

where

$$v = \frac{1}{(LC)^{\frac{1}{2}}} \tag{A-5}$$

is the propagation velocity.

$$\rho = (L/C)^{\frac{1}{2}} \tag{A-6}$$

is the characteristic impedance of line.

Applying A-3 and A-4 at the boundaries, x=0 and x=1 we obtain:

$$V(1,s) + \rho I \, (1,s) = V(0,s) \, e^{\frac{-sL}{v}} + \rho I \, (0,s) \, e^{\frac{-sL}{v}} \tag{A-7}$$

$$V(0,s) - \rho I\ (0,s) = V(1,s)\ e^{\frac{-sL}{v}} - \rho I(1,s)\ e^{\frac{-sL}{v}} \qquad \text{(A-8)}$$

Letting

$$T = \frac{1}{v}$$

Then we get in the time domain

$$V(1,t) + \rho i(1,t) = V(0,\ t-T) + \rho i(0,\ t-T) \qquad \text{(A-9)}$$

$$V(0,t) - \rho i(0,t) = V(1,\ t-T) - \rho i(1,\ t-T) \qquad \text{(A-10)}$$

It should be noted that the classical transmission line analysis adopted here assumes the following:

1. The transverse electric field components in the conductor are negligible compared with the axial. This is equivalent to the assumption that displacement currents in the conductor are negligible compared to conduction currents. This is stated mathematically as $\frac{\sigma}{\omega\varepsilon} >> 1$.

2. The axial electric field components in the dielectric are small compared with the transverse. This is equivalent to the assumption that the characteristic impedance of the dielectric is much greater than the skin effect surface resistivity of the conductor or $\frac{R_s}{\eta} << 1$.

These inequalities are nearly always satisfied by the materials of common transmission lines, but if they are not, one must examine critically any results predicted by the usual transmission line equations. Also, the longer the duration of the input current function, the better these inequalities are met.

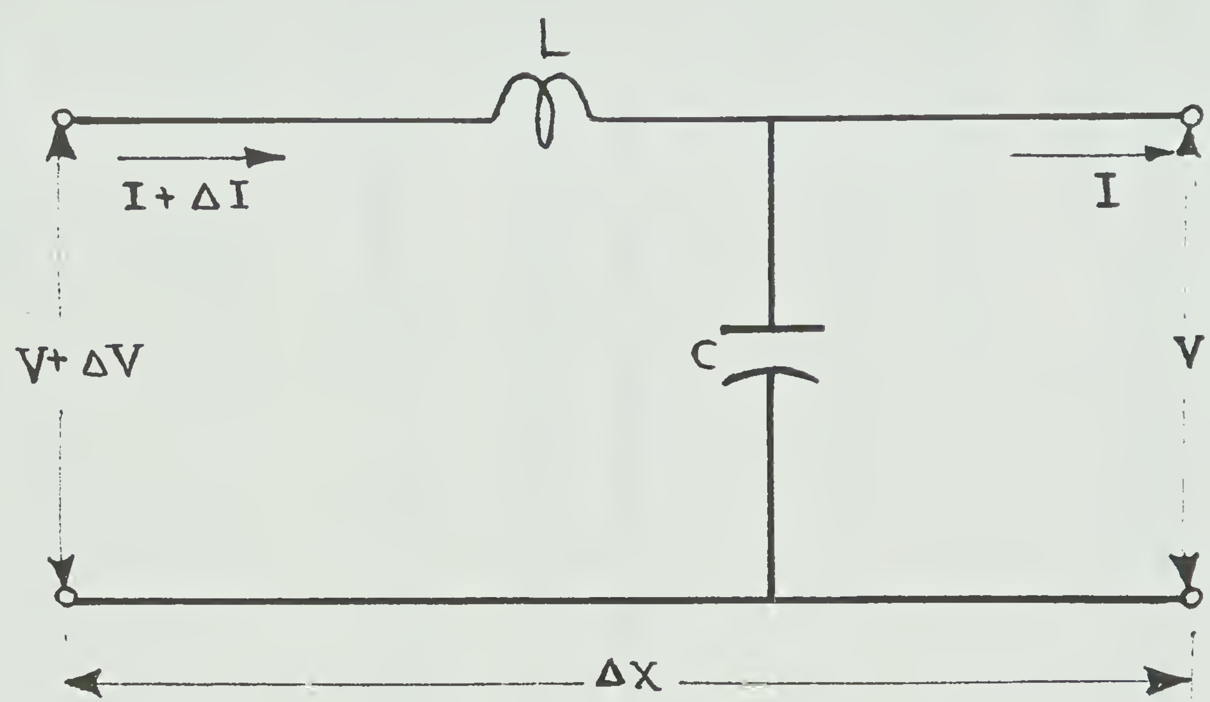

Fig. A-1 A Uniform Losseless Transmission Line

APPENDIX B

DERIVATION OF EQUATION (3-50)

Consider an incremental length Δx at a distance x from the sending end of a uniform lossy transmission line with length h. This incremental length is shown in fig. B-1, then we can write the following equations :

$$-\frac{\partial V}{\partial x} = (sL + R)\ I \qquad \text{(B-1)}$$

$$-\frac{\partial I}{\partial x} = (sC + G)\ V \qquad \text{(B-2)}$$

where R, L, C and G are resistance, inductance, capacitance and conductance per unit length respectively, s is the Laplace operator.

Partial differentiation of (B-1) and (B-2) w.r.t. x yields :

$$-\frac{\partial^2 V}{\partial x^2} = (sL + R)\ \frac{\partial I}{\partial x} \qquad \text{(B-3)}$$

$$-\frac{\partial^2 I}{\partial x^2} = (sC + G)\ \frac{\partial V}{\partial x} \qquad \text{(B-4)}$$

Substituting for $\frac{\partial I}{\partial x}$ and $\frac{\partial V}{\partial x}$ in (B-3) and (B-4) from (B-2) and (B-1)

respectively yields

$$\frac{\partial^2 V}{\partial x^2} = (sL + R)\ (sC + G)\ V \qquad \text{(B-5)}$$

$$\frac{\partial^2 I}{\partial x^2} = (sL + R)\ (sC + G)\ I \qquad \text{(B-6)}$$

Let

$$\gamma^2\ (s) = (sL + R)\ (sC + G) \qquad \text{(B-7)}$$

Then (B-5) and (B-6) can be rewritten as :

$$\frac{\partial^2 V}{\partial x^2} = \gamma^2\ (s)\ V \qquad \text{(B-8)}$$

$$\frac{\partial^2 I}{\partial x^2} = \gamma^2\ (s)\ I \qquad \text{(B-9)}$$

The solution to (B-8) is

$$V(x,s) = F(s)\ e^{-\gamma x} + H(s)\ e^{\gamma x} \qquad \text{(B-10)}$$

where F(s) and H(s) are functions to be determined such that boundary conditions along the line are satisfied. Using (B-10) and (B-1) we get for I(x,s) :

$$I(x,s) = \frac{\gamma(s)}{sL+R}\ [F(s)\ e^{-\gamma x} - H(s)\ e^{\gamma x}] \qquad \text{(B-11)}$$

In (B-10) and (B-11), forward propagating components of voltage and current are the ones having negative exponents, and

backward propagating components are those with positive exponents.

Thus if suffix f denotes forward components then we have

$$V_f(x,s) = F(s)\ e^{-\gamma x} \tag{B-12}$$

$$I_f(x,s) = \frac{\gamma(s)}{sL+R}\ F(s)\ e^{-\gamma x} \tag{B-13}$$

now at x = 0 we have

$$V_f(0,s) = F(s) \tag{B-14}$$

$$I_f(0,s) = \frac{\gamma(s)}{sL+R}\ F(s) \tag{B-15}$$

Thus (B-12) and (B-13) rewritten as

$$V_f(x,s) = V_f(0,s)\ e^{-\gamma x} \tag{B-16}$$

$$I_f(x,s) = I_f(0,s)\ e^{-\gamma x} \tag{B-17}$$

Now by definition of W(x,s) we have

$$W(x,s) = V_f(x,s) + \rho I_f(x,s) \tag{B-18}$$

then using (B-16) and (B-17) one gets

$$W(x,s) = W(0,s)\ e^{-\gamma x} \tag{B-19}$$

Let

$$W_K = W(0,s) \tag{B-20}$$

$$W_{K+1} = W(h,s) \tag{B-21}$$

then

$$W_{K+1} = W_K \exp[-\gamma h] \tag{B-22}$$

Now we have by (B-7)

$$\gamma(s) = [s^2 LC + (RC + GL)\ s + RG]^{\frac{1}{2}} \tag{B-23}$$

then

$$\gamma(s) = s\sqrt{LC}\ [1 + \frac{1}{s}\ (\frac{R}{L} + \frac{G}{C}) + \frac{RG}{s^2 LC}]^{\frac{1}{2}} \tag{B-24}$$

If the time t is small, then s is very large ($s \to \infty$ as $t \to 0$), then we can write

$$\gamma(s) = s\sqrt{LC}\ [1 + \frac{1}{s}\ (\frac{R}{L} + \frac{G}{C})]^{\frac{1}{2}} \tag{B-25}$$

Using the binomial theorem one gets

$$\gamma(s) = s\sqrt{LC}\ [1 + \frac{1}{2s}\ (\frac{R}{L} + \frac{G}{C})] \tag{B-26}$$

or

$$\gamma(s) = s\sqrt{LC} + \frac{1}{2}\ [\frac{R}{\rho} + G\ \rho] \tag{B-27}$$

$$h.\gamma(s) = s.h.\sqrt{LC} + \frac{1}{2}\ [\frac{Rh}{\rho} + G\ h.\rho] \tag{B-28}$$

Now by definition of Δt as the electrical length of the section with physical length h and letting

$$r = R.h \tag{B-29}$$

$$g = G.h \tag{B-30}$$

we have

$$h.\gamma(s) = \Delta t.s + \frac{1}{2} [\frac{r}{\rho} + g \rho] \tag{B-31}$$

Thus

$$W_{K+1} = W_K \exp [- \Delta t.s - \frac{1}{2} (\frac{r}{\rho} + g \rho)] \tag{B-32}$$

which is equation (3-50)

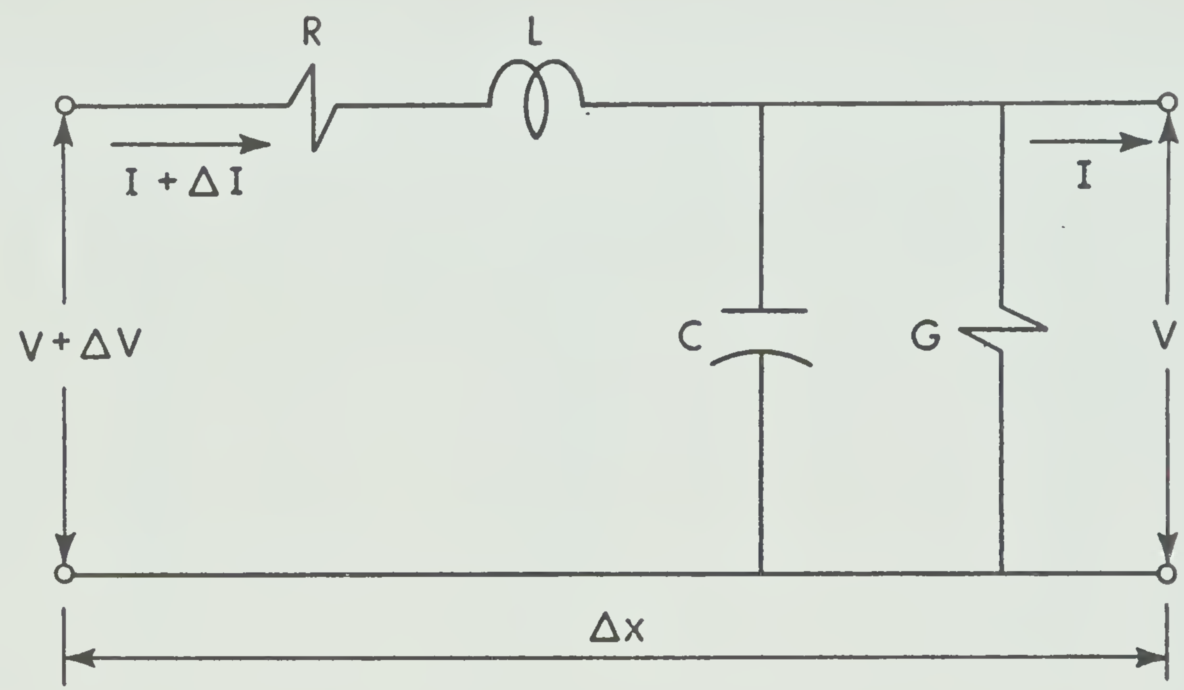

FIG. B-1 INCREMENTAL LENGTH OF A TRANSMISSION LINE

APPENDIX C

PROGRAM LISTING

```
              C      1....1....1....1....1....1....1
              C ********************  GENERAL TRANSMISSION LINE  *********************
              C ***********************  TRANSIENT ANALYSIS  *************************
              C
              C
              C      C(K),V(K)...OLD CURRENTS AND VOLTAGES AT THE SECTION BOUNDARIES
              C      CC(K),VV(K)...NEW VALUES CALCULATED FROM C(K),V(K)
              C      LINE DIVIDED INTO NN-1 SECTIONS
ISN 0002             DIMENSION YAB(1004),X11(1004),YBB(1004),X22(1004),YCB(1004),X33(10
                    204)
ISN 0003             DIMENSION C(1000),CC(1000),V(500),VV(500),X(1002,4),WORK(1024)
ISN 0004             DIMENSION FY(1001),Z(1001),DEL(1001),SY(1001),SZ(1002,3)
ISN 0005             DIMENSION RO(1001),YN(1002),Z1(1002)
ISN 0006             DIMENSION ZA(1002),COF1(501),COF2(501),COF5(501),COF6(501)
ISN 0007             DIMENSION ROO(1002,4),Y(1002,4),Y3(1002,4),Y4(1002)
ISN 0008             DIMENSION RAB(104),RBB(504),RCB(1004),Z11(104),Z22(504),Z33(1004)
ISN 0009             EXTERNAL FL,FC,FR,FG,SOUR
ISN 0010             COMMON/BGB/XL0,C0,R0,G0,XLE
ISN 0011             COMMON/SOS/XLE1,XLE2,XLE3,AMP,SLOPE
              C      WORK(N)....AUXILIARY VECTOR FOR PLOTTING
ISN 0012             CALL PLOTS (WORK(1),4096)
ISN 0013             CALL PLOT (5.0,5.0,-3)
              C      OUTPUT RESISTIVE LOAD=RR
ISN 0014             READ(5,901) RR,GG
ISN 0015         901 FORMAT(2E16.8)
ISN 0016             READ(5,902)XNN,XMEW
ISN 0017         902 FORMAT(2E16.8)
ISN 0018             READ(5,903) RESI,ROVAL,GVAL
ISN 0019         903 FORMAT(3E16.8)
ISN 0020             READ(5,904) XL0,C0,R0,G0
ISN 0021         904 FORMAT(4E16.8)
ISN 0022             WRITE(6,910) XL0,C0,R0,G0
ISN 0023         910 FORMAT(4E10.4)
ISN 0024             READ(5,905) XLE1,XLE2,XLE3,AMP,SLOPE
ISN 0025         905 FORMAT(5E12.4)
ISN 0026             IF (XMEW.EQ.0.0) GO TO 666
ISN 0028             ALFA=((RESI/ROVAL)+(GVAL*ROVAL))/2.
ISN 0029             XNN=ALFA*SQRT((ALFA)/(12.*XMEW))
ISN 0030             WRITE(6,906) RESI,ROVAL,GVAL,XMEW,ALFA,XNN
ISN 0031         906 FORMAT(1X,6E16.8)
ISN 0032             NN=XNN
ISN 0033             WRITE(6,907) NN
ISN 0034         907 FORMAT(30X,1I8)
ISN 0035             IF (NN.LT.10) NN=10
ISN 0037             IF (NN.GT.1000) GO TO 777
ISN 0039         666 FY(1)=0.00
ISN 0040             DO 1 K1=2,1001
ISN 0041             DEL(1)=SQRT((FL(0.0))*(FC(0.0)))
ISN 0042             XK=K1
ISN 0043             Z(K1)=(XK-1.)*0.001
ISN 0044             ZK=Z(K1)
ISN 0045             DEL(K1)=SQRT((FL(ZK))*(FC(ZK)))
ISN 0046           1 FY(K1)=FY(K1-1)+0.5*(DEL(K1)+DEL(K1-1))
ISN 0047             TOU=FY(1001)-FY(1)
```

```
ISN 0048          DO 2 IX=1,1000
ISN 0049          IX1=IX+1
ISN 0050        2 YN(IX)=FY(IX1)/TOU
ISN 0051          DO 3 IZ=1,1000
ISN 0052          IZ1=IZ+1
ISN 0053        3 Z1(IZ)=Z(IZ1)
ISN 0054          DO 7 INDEX=1,3
ISN 0055          IF ( INDEX .EQ. 2 ) XNN=1.5*XNN
ISN 0057          IF ( INDEX .EQ. 3 ) XNN=(4.0*XNN)/3.0
ISN 0059       10 DELT=TOU/XNN
ISN 0060          NN=XNN
ISN 0061          NP=NN-1
ISN 0062          DO 4 J1=1,NP
ISN 0063          SZ(1,INDEX)=0.0
ISN 0064          XJ=J1
ISN 0065          SY(J1)=XJ*DELT
ISN 0066          DO 4 K2=1,1000
ISN 0067          IF(SY(J1) .GT. FY(K2+1)) GO TO 4
ISN 0069          IF(SY(J1) .LT. FY(K2+1) .AND. SY(J1).GT. FY(K2)) Y1=FY(K2)
ISN 0071          IF(SY(J1) .LT. FY(K2+1) .AND. SY(J1).GT. FY(K2)) Y6=FY(K2+1)
ISN 0073          IF(SY(J1) .LT. FY(K2+1) .AND. SY(J1).GT. FY(K2)) KNN=K2
ISN 0075          J111=J1+1
ISN 0076          SZ(J111,INDEX)=Z(KNN)+(0.001*((SY(J1)-Y1)/(Y6-Y1)))
ISN 0077        4 CONTINUE
ISN 0078          SZ(NN+1,INDEX)=1.0
ISN 0079          NX=NN+1
ISN 0080          NY=NN+2
ISN 0081          DO 5 J5=1,NX
ISN 0082          XZJ=SZ(J5,INDEX)
ISN 0083          XL=FL(XZJ)
ISN 0084          XC=FC(XZJ)
ISN 0085          XR=FR(XZJ)
ISN 0086          XG=FG(XZJ)
ISN 0087          RO(J5)=SQRT(XL/XC)
ISN 0088          COF1(J5)=1.-(DELT*XG)/(2.*XC)
ISN 0089          COF2(J5)=RO(J5)-(DELT*XR)/(2.*(SQRT(XL*XC)))
ISN 0090          COF5(J5)=(2.)*(1.+(DELT*XG)/(2.*XC))
ISN 0091          COF6(J5)=(2.)*(RO(J5)+(DELT*XR)/(2.*(SQRT(XL*XC))))
ISN 0092        5 ROO(J5,INDEX)=RO(J5)
              C *** INITIAL CONDITIONS ON TRANSMISSION LINE ***
ISN 0093          DO 6 K3=1,NN
ISN 0094          C(K3)=0.0
ISN 0095        6 V(K3)=0.0
              C *** ITERATIVE CALCULATION, L=DISCRETE TIME ***
ISN 0096          NH=5*NN
ISN 0097          DO 7 L=1,NH
ISN 0098          XLE=L/XNN
ISN 0099          NP=NN-1
ISN 0100          DO 8 J=2,NP
ISN 0101          A=V(J-1)*COF1(J-1)+C(J-1)*COF2(J-1)
ISN 0102          B=V(J+1)*COF1(J+1)-C(J+1)*COF2(J+1)
ISN 0103          VV(J)=(A+B)/COF5(J)
ISN 0104          CC(J)=(A-B)/COF6(J)
ISN 0105        8 CONTINUE
              C *** BOUNDARY CONDITIONS ***
ISN 0106          XA=0.5*(COF5(1)+GG*COF6(1))
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ISN 0107          VV(1)=(0.5*(SOUR(XLE)*COF6(1))+(V(2)*COF1(2))-(C(2)*COF2(2)))/XA
ISN 0108          CC(1)=SOUR(XLE)-GG*VV(1)
ISN 0109          XB=0.5*(COF5(NN)+(COF6(NN)/RR))
ISN 0110          VV(NN)=((V(NP)*COF1(NP))+(C(NP)*COF2(NP)))/XB
ISN 0111          CC(NN)=VV(NN)/RR
            C *** OLD - NEW VALUES EXCHANGE ***
ISN 0112          DO 9 M=1,NN
ISN 0113          C(M)=CC(M)
ISN 0114        9 V(M)=VV(M)
ISN 0115      444 X(L,INDEX)=XLE
ISN 0116          Y(L,INDEX)=C(NN)
ISN 0117          Y3(L,INDEX)=SOUR(XLE)
            C     PRINT THE CURRENTS AT THE EVERY FIFTH TIME INSTANT:
ISN 0118          IF(L/5*5.EQ.L) WRITE (6,908) C(1),C(NN/4),C(NN/2),C(3*NN/4),C(NN)
ISN 0120      908 FORMAT (1X,6E20.7)
ISN 0121        7 CONTINUE
ISN 0122          RB11=0.0
ISN 0123          YB11=0.0
ISN 0124          NAX=(5*NN)/2
ISN 0125          DO 71 L=1,NAX
ISN 0126          YAB(L)=Y(L,1)
ISN 0127          YB11=AMAX1(YAB(L),YB11)
ISN 0128       71 X11(L)=X(L,1)
ISN 0129          ICN=NN/2
ISN 0130          DO 76 L=1,ICN
ISN 0131          RAB(L)=ROO(L,1)
ISN 0132          RB11=AMAX1(RAB(L),RB11)
ISN 0133          Z11(L)=SZ(L,1)
ISN 0134       76 CONTINUE
ISN 0135          YC11=0.0
ISN 0136          RC11=0.0
ISN 0137          NBX=(15*NN)/4
ISN 0138          DO 72 L=1,NBX
ISN 0139          YBB(L)=Y(L,2)
ISN 0140          YC11=AMAX1(YBB(L),YC11)
ISN 0141       72 X22(L)=X(L,2)
ISN 0142          JCN=(3*NN)/4
ISN 0143          DO 77 L=1,JCN
ISN 0144          RBB(L)=ROO(L,2)
ISN 0145          RC11=AMAX1(RBB(L),RC11)
ISN 0146          Z22(L)=SZ(L,2)
ISN 0147       77 CONTINUE
ISN 0148          YD11=0.0
ISN 0149          RD11=0.0
ISN 0150          NCX=5*NN
ISN 0151          DO 73 L=1,NCX
ISN 0152          YCB(L)=Y(L,3)
ISN 0153          YD11=AMAX1(YCB(L),YD11)
ISN 0154       73 X33(L)=X(L,3)
ISN 0155          KON=NN
ISN 0156          DO 78 L=1,KCN
ISN 0157          RCB(L)=ROO(L,3)
ISN 0158          RD11=AMAX1(RCB(L),RD11)
ISN 0159          Z33(L)=SZ(L,3)
ISN 0160       78 CONTINUE
ISN 0161          YBIG=AMAX1(YB11,YC11,YD11)
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ISN 0162          RBIG=AMAX1(RBI1,RC11,RD11)
ISN 0163          WRITE(6,74) YBIG,RBIG
ISN 0164       74 FORMAT(30X,2E16.8,/)
         C *** PLOTTING INSTRUCTIONS ***
ISN 0165          RAB(ION+1)=0.0
ISN 0166          RAB(ION+2)=RBIG/4.
ISN 0167          Z11(ION+1)=0.0
ISN 0168          Z11(ION+2)=0.2
ISN 0169          CALL AXIS(0.0,0.0,'RC',2,4.0,90.0,RAB(ION+1),RAB(ION+2),20.0)
ISN 0170          CALL AXIS(0.0,0.0,'ACTUAL LENTH',-13,5.0,0.0,Z11(ION+1),Z11(ION+2)
                 1,20.0)
ISN 0171          CALL LINE(Z11,RAB,ION,1,0,3)
ISN 0172          CALL PLOT(0.0,0.0,-3)
ISN 0173          RBB(JON+1)=0.0
ISN 0174          RBB(JON+2)=RBIG/4.
ISN 0175          Z22(JON+1)=0.0
ISN 0176          Z22(JON+2)=0.2
ISN 0177          CALL LINE(Z22,RBB,JON,1,0,3)
ISN 0178          CALL PLOT(0.0,0.0,-3)
ISN 0179          RCB(KON+1)=0.0
ISN 0180          RCB(KON+2)=RBIG/4.
ISN 0181          Z33(KON+1)=0.0
ISN 0182          Z33(KON+2)=0.2
ISN 0183          CALL LINE(Z33,RCB,KON,1,0,3)
ISN 0184          CALL PLOT(0.0,5.5,-3)
ISN 0185          CALL SCALE (YN,4.0,1000,1,20.0)
ISN 0186          CALL AXIS (0.0,0.0,'YN(IX)',6,4.0,90.0,YN(1001),YN(1002),20.0)
ISN 0187          CALL SCALE(Z1,5.0,1000,1,20.0)
ISN 0188          CALL AXIS (0.0,0.0,'ACTUAL LENGTH',-13,5.0,0.0,Z1(1001),Z1(1002),2
                 10.0)
ISN 0189          CALL LINE (Z1,YN,1000,1,0,3)
ISN 0190          CALL PLOT (12.0,0.0,-3)
ISN 0191          YAB(NAX+1)=0.0
ISN 0192          YAB(NAX+2)=YBIG/4.
ISN 0193          X11(NAX+1)=0.0
ISN 0194          X11(NAX+2)=1.0
ISN 0195          CALL AXIS (0.0,0.0,'LOAD CURRENT',12,4.0,90.0,YAB(NAX+1),YAB(NAX+2
                 1),20.0)
ISN 0196          CALL AXIS(0.0,0.0,'TIME',-4,5.0,0.0,X11(NAX+1),X11(NAX+2),20.0)
ISN 0197          CALL LINE(X11,YAB,NAX,1,0,3)
ISN 0198          CALL PLOT(0.0,0.0,-3)
ISN 0199          YBB(NBX+1)=0.0
ISN 0200          YBB(NBX+2)=YBIG/4.
ISN 0201          X22(NBX+1)=0.0
ISN 0202          X22(NBX+2)=1.0
ISN 0203          CALL LINE(X22,YBB,NBX,1,0,3)
ISN 0204          CALL PLOT(0.0,0.0,-3)
ISN 0205          YCB(NCX+1)=0.0
ISN 0206          YCB(NCX+2)=YBIG/4.
ISN 0207          X33(NCX+1)=0.0
ISN 0208          X33(NCX+2)=1.0
ISN 0209          CALL LINE(X33,YCB,NCX,1,0,3)
ISN 0210          CALL PLOT(0.0,5.5,-3)
ISN 0211          DO 600 I=1,1000
ISN 0212          Y4(I)=Y3(I,3)
ISN 0213      600 CONTINUE

ISN 0214          CALL SCALE(Y4,4.0,1000,1,20.0)
ISN 0215          CALL AXIS(0.0,0.0,'INPUT CURRENT',13,4.0,90.0,Y4(1001),Y4(1002),20
                 1.0)
ISN 0216          CALL AXIS(0.0,0.0,'TIME',-4,5.0,0.0,X33(NCX+1),X33(NCX+2),20.0)
ISN 0217          CALL LINE(X33,Y4,1000,1,0,3)
ISN 0218          CALL PLOT(10.0,7.0,-3)
ISN 0219          CALL PLOT(0.0,0.0,999)
ISN 0220          GO TO 888
ISN 0221      777 WRITE(6,909) NN
ISN 0222      909 FORMAT(30X,49H FOR THESE VALUES OF SYSTEM PARAMETERS WE GET NN=,I8
                 1/)
ISN 0223      888 STOP
ISN 0224          END
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```
ISN 0002          FUNCTION FC(DAN)
ISN 0003          COMMON/BOB/XL0,C0,R0,G0,XLE
ISN 0004          COMMON/SOS/XLE1,XLE2,XLE3,AMP,SLOPE
ISN 0005          FC=C0*EXP(-DAN)
ISN 0006          RETURN
ISN 0007          END

ISN 0002          FUNCTION FL(DAN)
ISN 0003          COMMON/BOB/XL0,C0,R0,G0,XLE
ISN 0004          COMMON/SOS/XLE1,XLE2,XLE3,AMP,SLOPE
ISN 0005          FL=XL0*(2.0+SIN(22.0*DAN/7.0))
ISN 0006          RETURN
ISN 0007          END

ISN 0002          FUNCTION FR(DAN)
ISN 0003          COMMON/BOB/XL0,C0,R0,G0,XLE
ISN 0004          COMMON/SOS/XLE1,XLE2,XLE3,AMP,SLOPE
ISN 0005          FR=R0*ALOG(1.0+DAN)
ISN 0006          RETURN
ISN 0007          END

ISN 0002          FUNCTION FG(DAN)
ISN 0003          COMMON/BOB/XL0,C0,R0,G0,XLE
ISN 0004          COMMON/SOS/XLE1,XLE2,XLE3,AMP,SLOPE
ISN 0005          FG=G0
ISN 0006          RETURN
ISN 0007          END

ISN 0002          FUNCTION SOUR(XAX)
ISN 0003          COMMON/BOB/XL0,C0,R0,G0,XLE
ISN 0004          COMMON/SOS/XLE1,XLE2,XLE3,AMP,SLOPE
ISN 0005          IF(XAX.LT.XLE1) SOUR=SLOPE*XAX
ISN 0007          IF(XAX.LT.XLE2.AND.XAX.GT.XLE1) SOUR=AMP
ISN 0009          IF(XAX.LT.XLE3.AND.XAX.GT.XLE2) SOUR=AMP*((XLE3-XAX)/(XLE3-XLE2))
ISN 0011          IF(XAX.GT.XLE3) SOUR=0.0
ISN 0013          RETURN
ISN 0014          END
```